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JUGFAE CRANE ERROR

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JULY 1989

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A concept called JUGFAE, JUG-contained Fuel-Air Explosive, was devised for clearing minefields in uncontested, rear areas. The concept prescribes a line of jugs set in by a crane standing on cleared ground. The jugs must be carefully spaced so that when they simultaneously burst, the fuel-air clouds can overlap and give one large explosion. The process is then repeated.

This report documents the mathematics of the error if the crane misspaces the jugs. Six different methods are presented. For a probability of 0.135 of mislaying a jug outside a tolerance circle whose diameter is no more than the cloud overlap expected, the probability of not detonating a 10-jug line is only 0.0404. The failure probability grows just less than directly proportionally to the number of jugs in the line.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

JUGFAE CRANE ERROR

1. INTRODUCTION

1.1 Purpose. A concept named JUGFAE, JUG-contained Fuel-Air Explosive, was disclosed (Ref. 1), tried (Ref. 2) and studied (Ref. 3) as a method for using fuel-air explosions to clear minefields in uncontested, rear areas. The latter efforts have shown that the concept has no fundamental flaw. Whether or not the concept is practical can only be found by field experience.

The analysis of failure needed mathematics that threatened to overrun the concept report (Ref. 3) and details were held back. This report's Appendix A supplies the details of the published analysis. Other appendices use separate, albeit agreeable, methods that were known to (Appendix B) or found afterwards by Umholtz (Appendix C,D).

The shortened work in Reference 3 dealt with the error involved in remote placement of numerous fueled jugs on the mined land. In Reference 2, the decision to use pre-emplaced jugs eliminated the remoting distraction from the crucial test of detonation transfer. That effort demonstrated that detonation wave transfer across overlapping clouds would take place.

Previous methods (Ref. 3) arduously solved the failure probability for a few lines with up to ten jugs placed on a line. The present results give the probability of failure with crane placement for all cases up to a 30-jug line. That length should include all lines which are practical to set up.

The reader may only want the results and to know that the six different methods produce the same low probability of failure for the JUGFAE concept. The mathematically inclined reader can profit by contrasting the methods. For instance, the method described in Appendix D is an achievement in conciseness and new attack direction that can only be appreciated after reading the earlier appendices. The mathematical efforts may be widely useful since the probability methods apply to a variety of problems.

1.2 Background. The JUGFAE process described crane-emplaced, fueled jugs spaced over mined land; one row of the process is depicted in Figure 1. If the carefully placed jugs were simultaneously burst, the numerous, growing fuel-air clouds would overlap and, by intent, one large explosion would be triggered an instant later. Susceptible land mines under the explosion would be neutralized. The "don't cross" line would be moved and the setup repeated. Advantages of the process are greater safety, completeness, and time saving over one-at-a-time shooting.

Overlap of the clouds is crucial to the JUGFAE process because a detonation wave cannot jump an air gap and resume detonating the rest of the clouds. An overlap region has to "chain together" every cloud to its neighbors in a line or the detonation wave dies out. When that happens, the resources utilized in setting up the line are partly wasted ("partial failure").

Reference 3 cites two reasons why clouds might not overlap: a "critically short cloud" and "crane error". The first reason was fully set out there and will just be briefly reviewed. The primary purpose of this report is to fully describe the mathematics of the crane error.

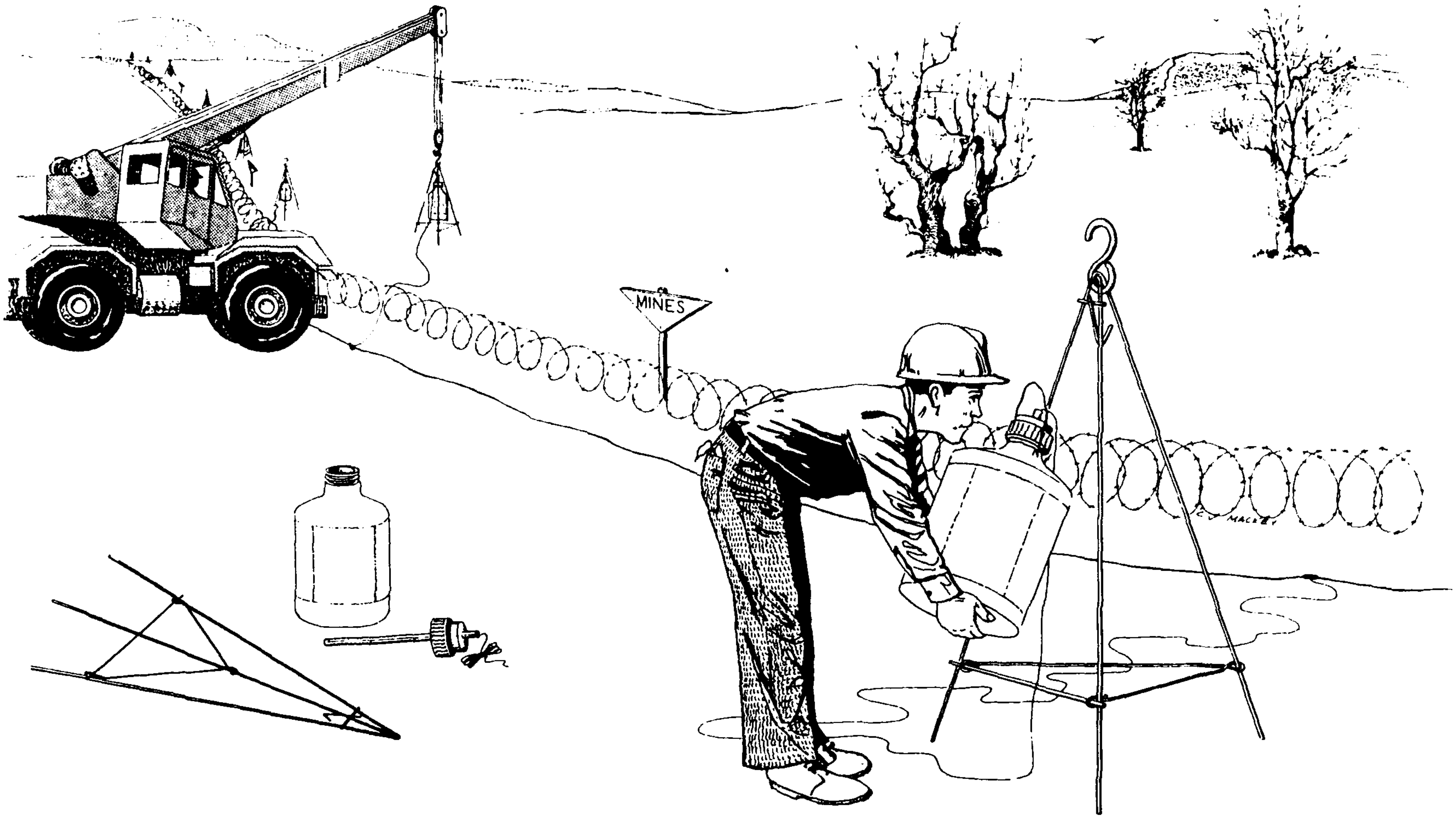

Figure 1. Laying in a Line of Jugs.

2. PARTIAL FAILURE

2.1 Critically Short Cloud. The first cause of partial failure is the inherent variability of the cloud growth process. Assume the jugs are perfectly placed on the correct separation marks. At the time when the whole cloud row is to be exploded (100 ms after the jugs burst), some clouds will overlap more and some less than is intended. However, only one disorder will cause partial failure. This occurs when one cloud out of the entire row just touches its two neighbor clouds. The cloud with the critically short radius (R_{crit}) causes a tangent geometry. The chaining together of the clouds (represented by circles) is lost. The situation is shown in Figure 2.

The explosion of the entire cloud row is caused by a single explosive charge, not one charge per cloud, for reasons of simplicity and time saving. The detonation wave started by the single charge cannot enter the tangent cloud and so the whole row is unexploded. A partial failure is recorded and the cost is partly-wasted time and effort that went into that setup.

The jug spacing (S) is seen in Figure 2 to be twice the cloud average radius, shortened by a distance that will statistically assure cloud overlap ($S = 2\bar{R} - P\bar{R}$). The amount of cloud overlap ($P\bar{R}$) needed depends directly on the variability of the clouds' radii (s) and on a probability coefficient (K) that represents with stated confidence what portion of all clouds will have radii that will exceed the critically short cloud's radius ($P\bar{R} = Ks$). Reference 3 found the jug spacing for different probabilities that a high proportion of clouds would overlap. For instance, clouds with a 6.23 m average radius and 0.25 m standard deviation need a jug separation of 11.3 m (19% overlap) to yield a probability of 99% that 95% of the clouds will overlap. (The probabilities do not depend on the number of jugs in the line.)

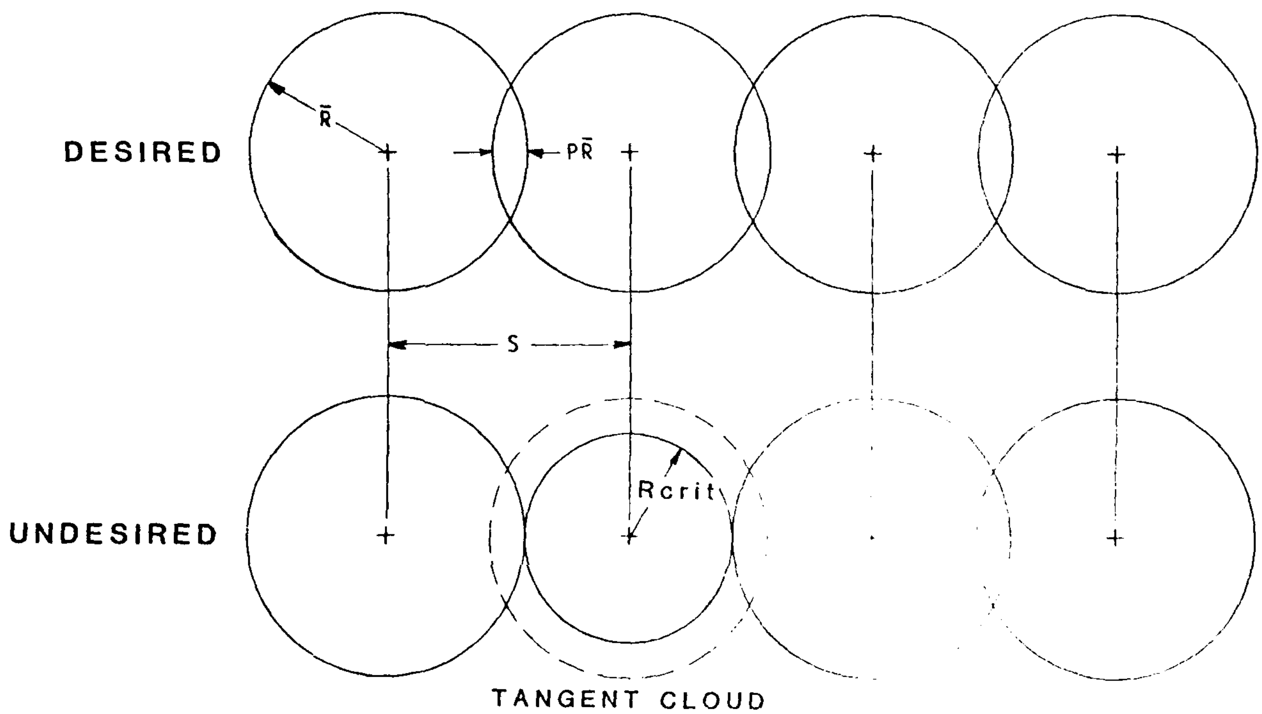

Figure 2. Critically Short Cloud.

2.2 Crane Placement of Jugs. Figure 3 shows four clouds from jugs that are perfectly placed and as they could look if the jugs were emplaced by a crane. All the clouds are drawn as circles with radius $\bar{R}$. The crookedness and mixed amount of overlap in the lower diagram are due to crane error. The crane sets a jug down within an error circle centered on the designated mark. (The error circle bounds the miss distance from the mark that we are willing to tolerate or must expect from crane emplacement.) For statistical reasons, the diameter of the crane error circle must be no more than the amount of cloud overlap. They are equal size in Figure 3 and, as it happens, a 29% cloud overlap has been drawn.

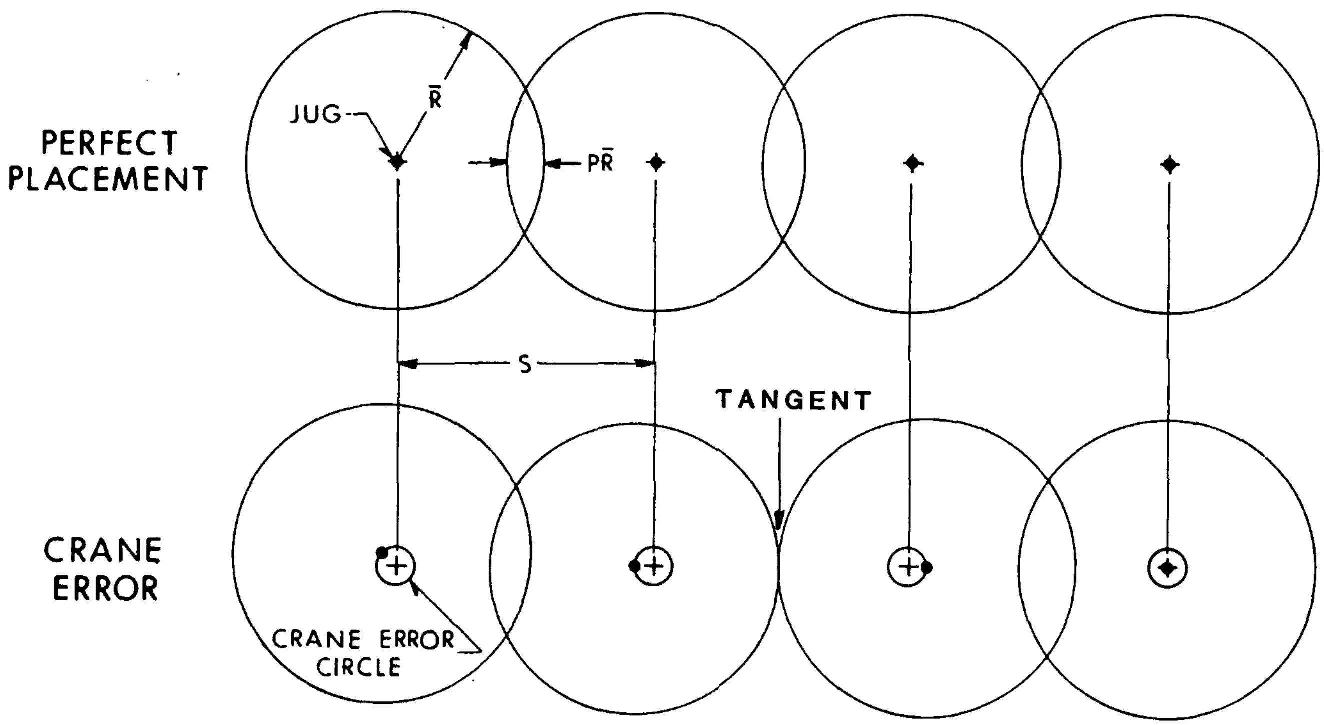

Figure 3. Crane Placement Error.

One part of Figure 3 also shows how the tangent geometry situation can arise from crane error. This very bad placement is enlarged in the lower part of Figure 4. The point illustrated is that the undesirable tangent geometry is now due to crane error, not the critically short (radius) cloud described in Section 2.1.

Let the diameter of the crane error circle be $2R_{crane}$. Let s_{crane} be the sample standard deviation of the miss distance of the jug from the mark. Using the sample standard deviation, s_{crane}, as an estimator of the population standard deviation, σ_{crane}, set $R_{crane} = 2\sigma_{crane}$, i.e., the maximum tolerable miss distance of setting up a jug is two standard deviations. For circular symmetry, and assuming the error around the mark is normally distributed (a uniform distribution is an alternative assumption), it is shown in Reference 4 that the probability of the event of interest, namely that the jug is outside the circle, is $p = 0.135$. Otherwise, the probability of one jug being set inside the circle is $q = 0.865$.

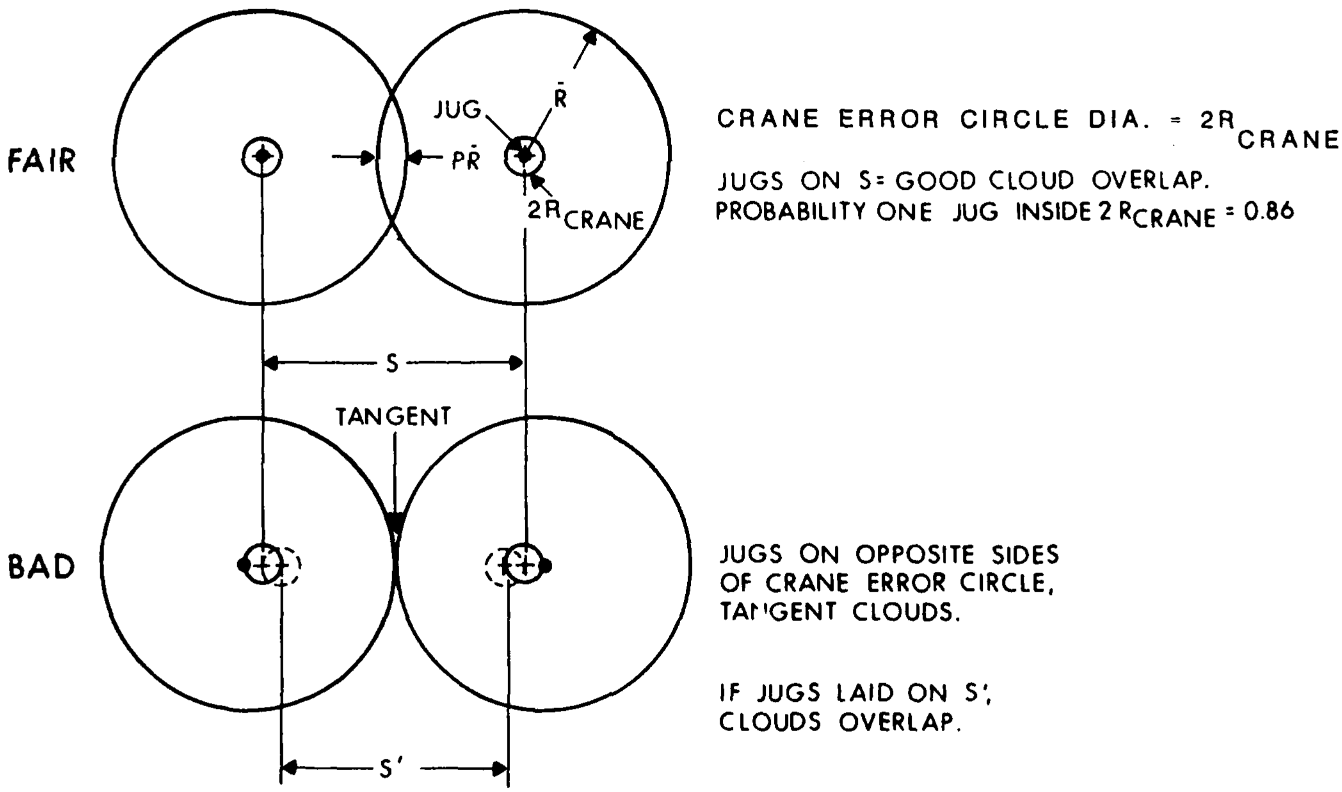

Figure 4. Worst Crane Placement.

An abstraction of Figure 4 shows that partial failure due to crane error proceeds only if several mistakes take place. First, the crane sets some jugs outside the permitted error circle and then these particular events also occur: the misset jugs are adjacent and on the left and right sides, respectively, of the crane error circles.* There is no partial failure if the jugs are set any of the other three possible ways that they can be both adjacent and mislaid. All the possibilities are shown in Figure 5.

Note that the probability of failure, given that two adjacent jugs are mislaid, is 1/4. These four combinations are basic to the analysis of crane error. The idea illustrated by Figure 5 is used throughout this report.

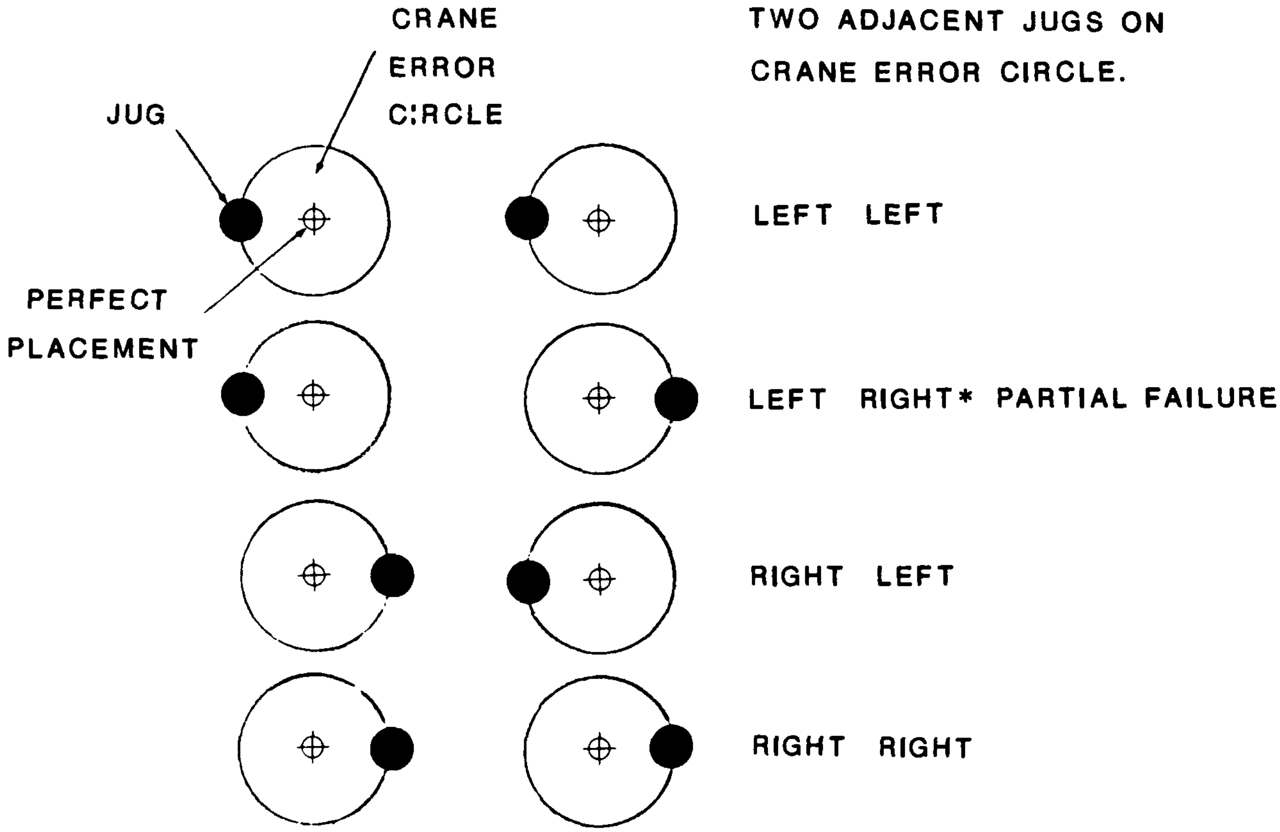

Figure 5. Misplacements of Adjacent Jugs.

* Left and right are just convenient names if one is next to the crane, watching it set in jugs. Left and right do not refer at all to which way the detonation wave is moving.

3. SUMMARY ANALYSES OF CRANE ERROR

3.1 Diagrams. The original, unpublished solution to the crane error problem is due to J. Richard Moore, Ballistic Research Laboratory, retired. It depended on conditional probability and seldom used mathematics of the probability of runs. The arcane run portion of the solution was replaced in Reference 3 with the diagrammatic method of counting the space of events. Although results for longer lines were given, the only worked-out example was for a 2-jug line. The diagrammatic method is shown for several cases in Appendix A; Moore's run solution is in Appendix B.

The basic equation used to calculate failure probability is given in Appendices A and B. It is

$$P(\text{failure}) = \sum_{i=1}^{n} P(f|i \text{ out})P(i \text{ out}). \tag{1}$$

To calculate P(failure) using Equation 1, take the probability of one jug of the line of n being out [P(1 out)] and multiply by the probability of failure for that arrangement, i.e., of one jug being out of the crane error circle [$P(f|1)$]; take the probability of two jugs of the line of n being out [P(2 out)] and multiply by the probability of failure for that arrangement, i.e., of two jugs being out of the crane error circle [$P(f|2)$]. Continue to form the product of these two terms all the way through the case of n jugs being mislaid. Add all the products and the sum is the probability of failure for an n-jug line.

The method presented in Appendix A is to make individual diagrams of one jug mislaid of the n-jug line, two jugs mislaid of the n-jug line, ... , n jugs mislaid of the n-jug line. Since the line cannot fail with only one jug out, i.e., $P(f|1 \text{ out}) = 0$, no diagram needs to be drawn. For two jugs out, all the diagrams must be drawn. A diagram with two adjacent jugs mislaid must be broken down further because only a left-right misplacement on the crane error circle leads to detonation failure (Section 2.2). The diagrams are counted for failure combinations and the count quickly gives the term $P(f|i \text{ out})$. Since the number of diagrams grows exponentially, e.g., a 6-jug line needs 2**6 or 64 diagrams, another technique is needed to find $P(f|i \geq 6)$.

3.2 Runs of Jugs. The method presented in Appendix B starts with Equation 1 but differs from Appendix A by finding $P(f|i \text{ out})$ utilizing run theory instead of diagrams. (The term is calculated by Equation B2 in a fashion similar to Equation 1). Equation B2 treats i jugs being out of place as meaning runs of misplaced jugs have formed. A run is an unbroken sequence. One mislaid jug is a run of one. Two mislaid jugs could mean two adjacent jugs are mislaid or two jugs anywhere on the line are mislaid; the former is one run of length two and the latter is two runs of length one. Each run that can be formed for i jugs being mislaid has a probability of occurrence that can be calculated using run theory, (see Equation B3). Each run also has a probability of failure, e.g., probability of failure of one run of length two is 1/4 as shown in Figure 5. The two probabilities are multiplied; all runs with mislaid jugs amounting to i are so treated and the products are summed to give $P(f|i \text{ out})$. The term P(i out) is found by the binomial distribution (Equation A3) from which Equation 1 is then computed. The theory of runs is clarified in Appendix B. Approximations are introduced to save labor, however, and cause a misleading indication that the probability of failure levels off after six or ten jugs; it actually rises almost linearly with jug number. We have not gone past 30 jugs, but obviously, for some large n, failure of the line is almost certain. Exact calculations are reported in Appendices C and D.

3.3 Runs of Arrangements. Moore's run method combined the conditional probability of a jug being misplaced with the probability of a run of such jugs. Conditional probability is not used in Appendix C but counting of runs is. The general idea is as follows. All possible runs of misplaced jugs are listed for an n-jug line. The product of three factors determines the probability of failure of a run. All the run probabilities are summed to yield the probability of failure with an n-jug line. Examples for 3- and 5-jug lines are given in Appendix C.

The idea in more detail is as follows. An n-jug line is regarded in Appendix C as an arrangement of jugs, whether outside or inside of the error circle. If all jugs are correctly placed, this arrangement is described as one run of length n. There is a probability of that arrangement happening and there is only one such run possible. The maximum number of runs is the integer part of (n+1)/2 for odd lines and n/2 for even lines, e.g., a 5-jug line has a maximum of three runs. (The maximum number of runs occurs in the jug arrangement where one end jug is out, the next jug is in, the next out, ..., and the other end jug is out.) Each run can occur in different arrangements along the line, e.g., a run of one in a 5-jug line can occur in five ways. In Appendix C, all runs from one to the maximum and all arrangements having that number of runs are found. The number of arrangements can be diagrammed or calculated by Equation C2. The probability of getting so many jugs out-and-in, Equation C1, the probability of failure of a particular run, Equations C4 and C5, and the number of arrangements possible, Equation C2, are multiplied together and the individual run probabilities are summed to get the probability of failure with an n-jug line, Equation C7. The procedure is programmed in "JUGRUN".

3.4 The Bad Event. The solution of the crane error problem described in Appendix D is distinctly different from the related approaches of the three previous appendices. The solution does not use either run theory or conditional probability. Instead it recognizes the bad event of Figure 4 as one that can arise n-1 times in an n-jug line. The bad event E occurs when adjacent jugs are set on the left side and the right side of their respective crane error circles. Let event E_1 be so setting jug 1 and jug 2; event E_2 is so setting jug 2 and jug 3; ... ; event E_{n-1} is so setting jug (n-1) and jug n. Failure (partial detonation of the n-jug line) will occur if at least one of the (n-1) events is realized.

$$P(\text{failure}) = P(E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_{n-1}), \qquad (2)$$

where "$\cup$" symbolizes "or".

The general expansion of Equation 2 is given in Appendix D and is performed two ways, leading to two dissimilar equations, D5 and D25, for the probability of failure. The two ways are indicated simply by the expansion of the union of any two sets A and B,

$$A \cup B = A + B - (A \cap B), \qquad (3)$$

$$A \cup B = A \quad (B \cap \bar{A}), \qquad (4)$$

where "$\cap$" symbolizes "and", "$\bar{A}$" symbolizes "not A".
Venn diagrams representing this union are shown in Appendix D. An important fact used for simplification in both approaches is that the intersection of adjacent events yields the null set because they cannot occur. For instance, in order for $E_1 \cup E_2$ to occur, it would require jug 2 to be both too far right and too far left.

The full expansion of Equation 2, in the manner of Equation 3, is presented in Appendix D.1. Most of the terms of Equation D2 will go to zero because of the intersection of mutually exclusive events. This simplification will be illustrated by the example of a 5-jug line. The remaining terms for an n-jug line are represented by Equation 5. To avoid calculating a factorial term, a recursive formula is given in Equation D7 that is better suited to computation. The procedure is programmed in "JUGCOM".

$$P(\text{failure}) = \sum_{i=1}^{\lfloor n/2 \rfloor} (-1)^{i+1} \binom{n-i}{i} p'^{\,i}, \tag{5}$$

where,

n = number of jugs in a row,
$\lfloor n/2 \rfloor$ = the integer part of n/2,
p = probability of an individual jug being outside error circle,
$p/2$ = probability of an individual jug being too far left OR
= probability of an individual jug being too far right,
$p' = (p/2)^2$ = probability of one jug being too far left and the next jug being too far right (by independence of placements of jugs) OR
= probability of a particular pair of consecutive jugs causing failure.

The full expansion of Equation 2, in the manner of Equation 4, is in Appendix D.2. A 2- and a 3-jug line are simply expressed, but a 4-jug line requires numerous set manipulations before arriving at the probability of failure as the sum of the failure probabilities for the 3-jug line and a constant times the failure probability of the 2-jug line (Equation D18). The identical form is given for the n-jug line by Equation 6. The procedure is programmed in "JUGREC".

$$P(\text{failure}) = P(\text{fail w/n-1 jugs}) + p'[1 - P(\text{fail w/n-2 jugs})]. \tag{6}$$

In Appendix D.3, an approximate formula for the probability of failure is given by Equation D30. The derivation starts with the general Equation 2, but wrongly assumes that the adjacent E events can occur. The deliberate error is made in Equation D28. The approximation is quite good for any jug line length and using probabilities of missetting one jug around $p \simeq 0.135$.

$$P(\text{failure}) \simeq 1 - [1 - (p/2)^2]^{n-1} . \tag{7}$$

4. RESULTS

The analyses yielded the probability of failure to detonate overlapped clouds from a row of jugs set in by a crane. Table 1 was calculated with a probability of missetting one jug of 0.135, for reasons set forth in Section 2.2, and the failure probability for up to a 30-jug row is calculated. Table 2 presents the failure probability of the row of jugs for other probabilities of missetting one jug around the value of 0.135. Figure 6 is a linear fit of the data in Table 2.

TABLE 1. Probability of Failure Due to Crane Error.

Number of Jugs in Row	Probability of Failure Due to Crane Error (*)
1	0.0000
2	.0046
3	.0091
4	.0136
5	.0182
6	.0227
7	.0271
8	.0316
9	.0360
10	.0404
11	.0448
12	.0492
13	.0535
14	.0579
15	.0622
16	.0665
17	.0708
18	.0750
19	.0792
20	.0835
21	.0877
22	.0918
23	.0960
24	.1001
25	.1042
26	.1083
27	.1124
28	.1165
29	.1205
30	.1246

(*) The crane is permitted to set the jug off the ideal mark but within an error circle whose radius is equivalent to half the cloud overlap distance, $\overline{PR}$. The probability of any individual jug being outside the circle is 0.135. The probability of failure due to a short radius cloud is about 0.01 (by Section 2.1 and Reference 3). The total probability of failure due to either a short radius cloud or crane error is found by adding 0.01 to the numbers in column two.

TABLE 2. JUGFAE Sensitivity.

Jugs	Probability of Missetting One Jug, (p).			
	p = .1	p = .135	p = .15	p = .2
n	Probability of Failure for n-Jug Line			
2	0.0025	0.0046	0.0056	0.0100
3	.0050	.0091	.0113	.0200
4	.0075	.0136	.0168	.0299
5	.0100	.0182	.0224	.0397
6	.0125	.0227	.0279	.0494
7	.0149	.0271	.0334	.0590
8	.0174	.0316	.0389	.0685
9	.0199	.0360	.0443	.0779
10	.0223	.0404	.0497	.0872
11	.0248	.0448	.0551	.0965
12	.0272	.0492	.0605	.1056
13	.0297	.0535	.0658	.1146
14	.0321	.0579	.0711	.1236
15	.0345	.0622	.0763	.1324
16	.0369	.0665	.0815	.1412
17	.0393	.0708	.0867	.1499
18	.0418	.0750	.0919	.1584
19	.0442	.0792	.0970	.1669
20	.0466	.0835	.1022	.1754
21	.0489	.0877	.1072	.1837
22	.0513	.0918	.1123	.1919
23	.0537	.0960	.1173	.2001
24	.0561	.1001	.1223	.2082
25	.0584	.1042	.1273	.2162
26	.0608	.1083	.1322	.2241
27	.0632	.1124	.1371	.2319
28	.0655	.1165	.1420	.2397
29	.0678	.1205	.1468	.2474
30	.0702	.1246	.1517	.2550

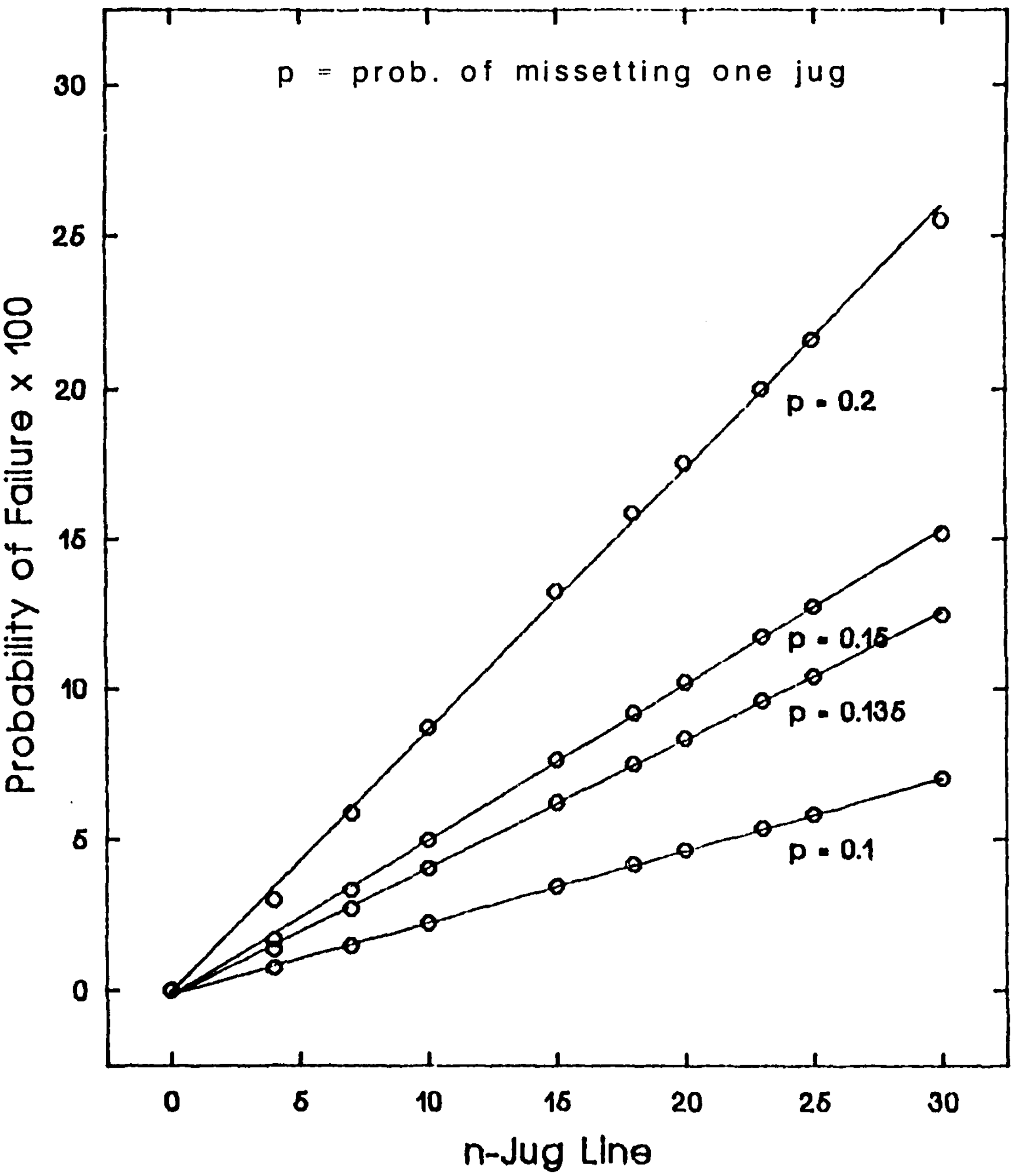

Figure 6. Linear Plot of Failure to Detonate the Line.

5. CONCLUSION

Using the JUGFAE concept, an uncontested minefield could be cleared, a line at a time, by one detonation of overlapping fuel-air clouds. The clouds are produced from fuel jugs set down by a crane operating parallel to and alongside the edge of the minefield. If the jugs could be ideally spaced, the vagaries in cloud size would not prevent their necessary overlap. The crane will, however, set each jug around the ideal mark. This crane error can prevent cloud overlap unless each jug is set down within the allowable error circle around the ideal mark.

This report presents different analyses of the probability of failure, caused by crane error, to detonate clouds from a line of jugs. The methods used include numeration of event space, conditional probability, run theory and set theory, either separately or combined. Set theory proved to be much shorter, both conceptually and computationally, than the other methods.

One result is that given an expected probability of 0.135 of missetting one jug, the probability of not detonating an entire 10-jug line is 0.0404. Furthermore, the probability of failure is a little less than proportional to the number of jugs in the line. The major result is that crane error results in a manageably small probability of failure to detonate the entire line.

The extensive analysis supports the conclusion that there is no basic flaw with the JUGFAE concept. However, the concept's operational feasibility has yet to be tested.

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APPENDIX A

DIAGRAMS OF CRANE ERROR FOR THREE-, FOUR-, AND FIVE-JUG LINES

A.1 Explanation. The harm in setting the jugs off the spacing mark is the likely nonoverlap of some clouds and subsequent failure to propagate the detonation. The probability of failure of a two-jug line was analyzed completely in Reference 3. Diagrams showing all possible placements were used. The method is extended here to more lines. Definitions of events leading to failure and the probability of failure equation are reviewed. The calculations and respective diagrams are listed for three-, four-, and five-jug lines. The diagrams use (+) for a jug on the mark and (0) for a jug set outside the permitted error circle around the mark. A failure situation (*) occurs when adjacent jugs are placed on the left (L) and right (R) sides of their respective error circles, producing tangent clouds. (See Figure 4.)

For a cloud line detonation failure to occur, three events must take place during jug placement:

E_0 = the crane puts 2 or more jugs outside their permitted error circles,

E_1 = given that E_0 has occurred, at least two outside jugs are adjacent,

E_2 = given that E_1 has occurred, adjacent jugs are on the left and right sides, respectively, of their error circles.

Failure to transmit detonation with crane placement of jugs can be restated as: failure occurs if, given that E_0 has occurred, then E_1 and E_2 occur.

The probability of failure P(failure) can be written

$$P(\text{failure}) = \sum_{i=0}^{n} P_i(E_0)\, P_i(E_2, E_1 | E_0). \qquad \text{(A1)}$$

The first term is found by the binomial distribution,

$$P_i(E_0) = C_r^n\, p^r (1 - p)^{n-r}, \quad r = i,\ n = \text{total number of jugs} \qquad \text{(A2)}$$

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}, \quad n! = n(n-1)(n-2)\ldots(3)(2)(1). \qquad \text{(A3)}$$

The second term, a conditional probability term, is found using diagrams which will be listed later in this Appendix. Equation A1 will be expanded for the three cases to be discussed.

Three-jug line.

$$P(\text{failure}) = P(0\ \text{out})P(E_2,E_1|0\ \text{out}) + P(1\ \text{out})P(E_2,E_1|1\ \text{out}) + P(2\ \text{out})P(E_2,E_1|2\ \text{out}) + P(3\ \text{out})P(E_2,E_1|3\ \text{out}). \quad \text{(A4)}$$

From Section 2.2, the simple probability of mislaying one jug, i.e., setting it outside the crane error circle, is p = 0.135. By Equations A2 and A3,

$$P_0(E_0) = P(0\ \text{out}) = C^3_0 p^0 (1-p)^{3-0} = 1(.135)^0(.865)^3 = 0.6472$$

$$P_1(E_0) = P(1\ \text{out}) = C^3_1 p^1 (1-p)^{3-1} = 3(.135)^1(.865)^2 = 0.3030$$

$$P_2(E_0) = P(2\ \text{out}) = C^3_2 p^2 (1-p)^{3-2} = 3(.135)^2(.865)^1 = 0.0473 \quad \text{(A5)}$$

$$P_3(E_0) = P(3\ \text{out}) = C^3_3 p^3 (1-p)^{3-3} = 1(.135)^3(.865)^0 = 0.00246$$

The conditional probabilities are brought forward from the 3-jug diagram.

$$P(\text{failure}) = 0.6472(0) + 0.3030(0) + 0.0473(1/6) + 0.00246(4/8),$$

$$P(\text{failure}) = 0.00911 \text{ for a 3-jug line as shown in Table 1.} \quad \text{(A6)}$$

Four-jug line.

$$P(\text{failure}) = P(0\ \text{out})P(E_2,E_1|0\ \text{out}) + \ldots + P(4\ \text{out})P(E_2,E_1|4\ \text{out}). \quad \text{(A7)}$$

$$P(0\ \text{out}) = C^4_0 p^0(1-p)^4 = 1(.135)^0(.865)^4 = 0.560$$

$$P(1\ \text{out}) = C^4_1 p^1(1-p)^3 = 4(.135)^1(.865)^3 = 0.349$$

$$P(2\ \text{out}) = C^4_2 p^2(1-p)^2 = 6(.135)^2(.865)^2 = 0.082 \quad \text{(A8)}$$

$$P(3\ \text{out}) = C^4_3 p^3(1-p)^1 = 4(.135)^3(.865)^1 = 0.0085$$

$$P(4\ \text{out}) = C^4_4 p^4(1-p)^0 = 1(.135)^4(.865)^0 = 0.00033$$

The conditional probabilities are brought forward from the 4-jug diagram.

$$P(\text{fail}) = 0.560(0) + 0.349(0) + 0.082(1/8) + 0.0085(12/32) + 0.00033(11/16),$$

$$P(\text{fail}) = 0.0136 \text{ for a 4-jug line as shown in Table 1.} \quad \text{(A9)}$$

Five-jug line.

$$P(\text{failure}) = P(0\text{ out})P(E_2,E_1|0\text{ out}) + \ldots + P(5\text{ out})P(E_2,E_1|5\text{ out}). \tag{A10}$$

$$
\begin{aligned}
P(0\text{ out}) &= C^5_0\, p^0 (1-p)^5 = 1(.135)^0(.865)^5 = 0.4842 \\
P(1\text{ out}) &= C^5_1\, p^1 (1-p)^4 = 5(.135)^1(.865)^4 = 0.3779 \\
P(2\text{ out}) &= C^5_2\, p^2 (1-p)^3 = 10(.135)^2(.865)^3 = 0.1179 \\
P(3\text{ out}) &= C^5_3\, p^3 (1-p)^2 = 10(.135)^3(.865)^2 = 0.0184 \\
P(4\text{ out}) &= C^5_4\, p^4 (1-p)^1 = 5(.135)^4(.865)^1 = 0.00144 \\
P(5\text{ out}) &= C^5_5\, p^5 (1-p)^0 = 1(.135)^5(.865)^0 = 0.00004
\end{aligned} \tag{A11}
$$

The conditional probabilities are brought forward from the 5-jug diagram.

$$P(\text{failure}) = 0.4842(0) + 0.3779(0) + 0.1179(1/10) + 0.0184(24/80) + 0.00144(45/80) + 0.00004(26/32).$$

$$P(\text{failure}) = 0.01815 \text{ for a 5-jug line as shown in Table 1.} \tag{A12}$$

A.2 Diagrams. All the arrangements of three-, four-, and five-jug lines are drawn in the way explained in Section A.1. The equations of Section A.1 help account for all diagrams. The number of arrangements is C^n_r and since each jug out (up to r) can be mislaid left and right, there are 2^r sub-pictures under each arrangement. Not all diagrams need to be drawn explicitly. The mirror images are redundant.* (With an arrangement read from left to right, its mirror image is the one read right to left. For example, + 0 0 0 and 0 0 0 + are mirror images; the probability of failure is 4/8 for each.)

A reason to draw all diagrams is that they directly inform a project officer of the probability of failure if he has knowledge of how many or which particular jugs are mislaid.

---

*Dixon, Lisa A., personal communication, Aberdeen Research Center, Aberdeen, Maryland, August 1988.

THREE-JUG LINE

no jugs out

$C^3_0 = 1$

+	+	+
		0/0

$P(E_2,E_1 \mid 0\ \text{out}) = 0$

$2^0 = 1$

one jug out

$C^3_1 = 3$

+	+	0		+	0	+		0	+	+
+	+	L		+	L	+		L	+	+
+	+	R		+	R	+		R	+	+
		0/0				0/0				0/0

$2^1 = 2$

$P(E_2,E_1 \mid 1\ \text{out}) = (0 + 0 + 0)/3 = 0$

two jugs out

$C^3_2 = 3$

+	0	0		0	+	0		0	0	+	
+	L	L		L	+	L		L	L	+	
+	L	R	*	L	+	R		L	R	+	*
+	R	L		R	+	L		R	L	+	
+	R	R		R	+	R		R	R	+	
		1/4				0				1/4	

$2^2 = 4$

$P(E_2,E_1 \mid 2\ \text{out}) = (1/4 + 0 + 1/4)/3 = 1/6$

three jugs out

$C^3_3 = 1$

0	0	0	
L	L	L	
L	L	R	*
L	R	L	*
L	R	R	*
R	L	L	
R	L	R	*
R	R	L	
R	R	R	
		4/8	

$2^3 = 8$

$P(E_2,E_1 \mid 3\ \text{out}) = 4/8$

* failure situation

FOUR-JUG LINE

no jugs out

$C^{4}_{0} = 1$

+ + + +
0

$2^{0} = 1$

$P(E_2, E_1 \mid 0 \text{ out}) = 0$

one jug out

$C^{4}_{1} = 4$

$2^{1} = 2$

+ + + 0	+ + 0 +	+ 0 + +	0 + + +
+ + + L	+ + L +	+ L + +	L + + +
+ + + R	+ + R +	+ R + +	R + + +
0	0	0	0

$P(E_2, E_1 \mid 1 \text{ out}) = (0 + 0 + 0 + 0)/4 = 0$

two jugs out

$C^{4}_{2} = 6$

$2^{2} = 4$

+ + 0 0	+ 0 0 +	0 0 + +	0 + + 0	0 + 0 +	+ 0 + 0
+ + L L	+ L L +	L L + +	L + + L	L + L +	+ L + L
+ + L R *	+ L R + *	L R + + *	L + + R	L + R +	+ L + R
+ + R L	+ R L +	R L + +	R + + L	R + L +	+ R + L
+ + R R	+ R R +	R R + +	R + + R	R + R +	+ R + R
1/4	1/4	1/4	0	0	0

$P(E_2, E_1 \mid 2 \text{ out}) = (1/4 + 1/4 + 1/4 + 0 + 0 + 0)/6 = 1/8$

three jugs out

$C^{4}_{3} = 4$

$2^{3} = 8$

+ 0 0 0	0 + 0 0	0 0 + 0	0 0 0 +
+ L L L	L + L L	L L + L	L L L +
+ L L R *	L + L R *	L L + R	L L R +*
+ L R L *	L + R L	L R + L *	L R L +*
+ L R R *	L + R R	L R + R *	L R R +*
+ R L L	R + L L	R L + L	R L L +
+ R L R *	R + L R *	R L + R	R L R +*
+ R R L	R + R L	R R + L	R R L +
+ R R R	R + R R	R R + R	R R R +
4/8	2/8	2/8	4/8

$P(E_2, E_1 \mid 3 \text{ out}) = (4/8 + 2/8 + 2/8 + 4/8)/4 = 12/32$

* failure situation

FOUR-JUG LINE (continued)

four jugs out

$C^4_4 = 1$

$2^4 = 16$

0	0	0	0		0	0	0	0		0	0	0	0		0	0	0	0
L	L	L	L		L	R	L	L *		R	L	L	L		R	R	L	L
L	L	L	R *		L	R	L	R *		R	L	L	R *		R	R	L	R*
L	L	R	L *		L	R	R	L *		R	L	R	L *		R	R	R	L
L	L	R	R *		L	R	R	R *		R	L	R	R *		R	R	R	R
																	11/16	

$P(E_2, E_1 \mid 4 \text{ out}) = 11/16$

FIVE-JUG LINE

no jugs out

$C^5_0 = 1$

$2^0 = 1$

+	+	+	+	+

$P(E_2, E_1 \mid 0 \text{ out}) = 0$

one jug out

$C^5_1 = 5$

$2^1 = 2$

+	+	+	+	0		+	+	+	0	+		+	+	0	+	+
+	+	+	+	L		+	+	+	L	+		+	+	L	+	+
+	+	+	+	R		+	+	+	R	+		+	+	R	+	+
				0						0						0

+	0	+	+	+		0	+	+	+	+
+	L	+	+	+		L	+	+	+	+
+	R	+	+	+		R	+	+	+	+
				0						0

$P(E_2, E_1 \mid 1 \text{ out}) = (0 + 0 + 0 + 0 + 0)/5 = 0$

two jugs out

$C^5_2 = 10$

$2^2 = 4$

+	+	+	0	0		+	+	0	+	0		+	0	+	+	0
+	+	+	L	L		+	+	L	+	L		+	L	+	+	L
+	+	+	L	R *		+	+	L	+	R		+	L	+	+	R
+	+	+	R	L		+	+	R	+	L		+	R	+	+	L
+	+	+	R	R		+	+	R	+	R		+	R	+	+	R
				1/4						0						0

* failure situation

FIVE-JUG LINE (continued)

two jugs out (continued)

0	+	+	+	0	
L	+	+	+	L	
L	+	+	+	R	
R	+	+	+	L	
R	+	+	+	R	0

+	+	0	0	+	
+	+	L	L	+	
+	+	L	R	+	*
+	+	R	L	+	
+	+	R	R	+	1/4

+	0	+	0	+	
+	L	+	L	+	
+	L	+	R	+	
+	R	+	L	+	
+	R	+	R	+	0

0	+	+	0	+	
L	+	+	L	+	
L	+	+	R	+	
R	+	+	L	+	
R	+	+	R	+	0

+	0	0	+	+	
+	L	L	+	+	
+	L	R	+	+	*
+	R	L	+	+	
+	R	R	+	+	1/4

0	+	0	+	+	
L	+	L	+	+	
L	+	R	+	+	
R	+	L	+	+	
R	+	R	+	+	0

0	0	+	+	+	
L	L	+	+	+	
L	R	+	+	+	*
R	L	+	+	+	
R	R	+	+	+	1/4

$P(E_2,E_1 | 2 \text{ out}) = (1/4 + 1/4 + 1/4 + 1/4)/10 = 1/10$

three jugs out

$C^5_3 = 10$

$2^3 = 8$

+	+	0	0	0	
+	+	L	L	L	
+	+	L	L	R	*
+	+	L	R	L	*
+	+	L	R	R	*
+	+	R	L	L	
+	+	R	L	R	*
+	+	R	R	L	
+	+	R	R	R	4/8

+	0	+	0	0	
+	L	+	L	L	
+	L	+	L	R	*
+	L	+	R	L	
+	L	+	R	R	
+	R	+	L	L	
+	R	+	L	R	*
+	R	+	R	L	
+	R	+	R	R	2/8

0	+	+	0	0	
L	+	+	L	L	
L	+	+	L	R	*
L	+	+	R	L	
L	+	+	R	R	
R	+	+	L	L	
R	+	+	L	R	*
R	+	+	R	L	
R	+	+	R	R	2/8

0	+	0	+	0	
L	+	L	+	L	
L	+	L	+	R	
L	+	R	+	L	
L	+	R	+	R	
R	+	L	+	L	
R	+	L	+	R	
R	+	R	+	L	
R	+	R	+	R	0

0	0	+	+	0	
L	L	+	+	L	
L	L	+	+	R	
L	R	+	+	L	*
L	R	+	+	R	*
R	L	+	+	L	
R	L	+	+	R	
R	R	+	+	L	
R	R	+	+	R	2/8

0	0	+	0	+	
L	L	+	L	+	
L	L	+	R	+	
L	R	+	L	+	*
L	R	+	R	+	*
R	L	+	L	+	
R	L	+	R	+	
R	R	+	L	+	
R	R	+	R	+	2/8

* failure situation

FIVE-JUG LINE (continued)

three jugs out (continued)

0	0	0	+	+	
L	L	L	+	+	
L	L	R	+	+	*
L	R	L	+	+	*
L	R	R	+	+	*
R	L	L	+	+	
R	L	R	+	+	*
R	R	L	+	+	
R	R	R	+	+	
					4/8

+	0	0	0	+	
+	L	L	L	+	
+	L	L	R	+	*
+	L	R	L	+	*
+	L	R	R	+	*
+	R	L	L	+	
+	R	L	R	+	*
+	R	R	L	+	
+	R	R	R	+	
					4/8

+	0	0	+	0	
+	L	L	+	L	
+	L	L	+	R	
+	L	R	+	L	*
+	L	R	+	R	*
+	R	L	+	L	
+	R	L	+	R	
+	R	R	+	L	
+	R	R	+	R	
					2/8

0	+	0	0	+	
L	+	L	L	+	
L	+	L	R	+	*
L	+	R	L	+	
L	+	R	R	+	
R	+	L	L	+	
R	+	L	R	+	*
R	+	R	L	+	
R	+	R	R	+	
					2/8

$$P(E_2, E_1 | 3 \text{ out}) = (4/8 + 2/8 + 2/8 + 0 + 2/8 + 2/8 + 4/8$$
$$+ 4/8 + 2/8 + 2/8)/10 = 24/80$$

* failure situation

four jugs out

$C^{5}_{4} = 5$

$2^{4} = 16$

+	0	0	0	0	
+	L	L	L	L	
+	L	L	L	R	*
+	L	L	R	L	*
+	L	L	R	R	*
+	L	R	L	L	*
+	L	R	L	R	*
+	L	R	R	L	*
+	L	R	R	R	*
+	R	L	L	L	
+	R	L	L	R	*
+	R	L	R	L	*
+	R	L	R	R	*
+	R	R	L	L	
+	R	R	L	R	*
+	R	R	R	L	
+	R	R	R	R	

11/16

0	+	0	0	0	
L	+	L	L	L	
L	+	L	L	R	*
L	+	L	R	L	*
L	+	L	R	R	*
L	+	R	L	L	
L	+	R	L	R	*
L	+	R	R	L	
L	+	R	R	R	
R	+	L	L	L	
R	+	L	L	R	*
R	+	L	R	L	*
R	+	L	R	R	*
R	+	R	L	L	
R	+	R	L	R	*
R	+	R	R	L	
R	+	R	R	R	

8/16

0	0	+	0	0	
L	L	+	L	L	
L	L	+	L	R	*
L	L	+	R	L	
L	L	+	R	R	
L	R	+	L	L	*
L	R	+	L	R	*
L	R	+	R	L	*
L	R	+	R	R	*
R	L	+	L	L	
R	L	+	L	R	*
R	L	+	R	L	
R	L	+	R	R	
R	R	+	L	L	
R	R	+	L	R	*
R	R	+	R	L	
R	R	+	R	R	

7/16

0	0	0	+	0	
L	L	L	+	L	
L	L	L	+	R	
L	L	R	+	L	*
L	L	R	+	R	*
L	R	L	+	L	*
L	R	L	+	R	*
L	R	R	+	L	*
L	R	R	+	R	*
R	L	L	+	L	
R	L	L	+	R	
R	L	R	+	L	*
R	L	R	+	R	*
R	R	L	+	L	
R	R	L	+	R	
R	R	R	+	L	
R	R	R	+	R	

8/16

0	0	0	0	+	
L	L	L	L	+	
L	L	L	R	+	*
L	L	R	L	+	*
L	L	R	R	+	*
L	R	L	L	+	*
L	R	L	R	+	*
L	R	R	L	+	*
L	R	R	R	+	*
R	L	L	L	+	
R	L	L	R	+	*
R	L	R	L	+	*
R	L	R	R	+	*
R	R	L	L	+	
R	R	L	R	+	*
R	R	R	L	+	
R	R	R	R	+	

11/16

$P(E_2, E_1 | 4 \text{ out}) = (11/16 + 8/16 + 7/16 + 8/16 + 11/16)/5 = 45/80$

* failure situation

FIVE-JUG LINE (continued)

five jugs out

$C^5_5 = 1$

$2^5 = 32$

0	0	0	0	0	
L	L	L	L	L	
L	L	L	L	R	*
L	L	L	R	L	*
L	L	L	R	R	*
L	L	R	L	L	*
L	L	R	L	R	*
L	L	R	R	L	*
L	L	R	R	R	*
L	R	L	L	L	*
L	R	L	L	R	*
L	R	L	R	L	*
L	R	L	R	R	*
L	R	R	L	L	*
L	R	R	L	R	*
L	R	R	R	L	*
L	R	R	R	R	*
R	L	L	L	L	
R	L	L	L	R	*
R	L	L	R	L	*
R	L	L	R	R	*
R	L	R	L	L	*
R	L	R	L	R	*
R	L	R	R	L	*
R	L	R	R	R	*
R	R	L	L	L	
R	R	L	L	R	*
R	R	L	R	L	*
R	R	L	R	R	*
R	R	R	L	L	
R	R	R	L	R	*
R	R	R	R	L	
R	R	R	R	R	
					26/32

$P(E_2, E_1 | 5 \text{ out}) = 26/32$

* failure situation

APPENDIX B

THEORY OF RUNS APPLIED TO CRANE ERROR FOR THREE- AND TEN-JUG LINES

The probability of failure due to jug placement error by the crane is

$$P(\text{failure}) = \sum_{i=0}^{n} P_i(E_0)P_i(E_2,E_1 \mid E_0), \tag{B1}$$

where the E events were defined in Appendix A. The first term is the probability of the crane placing i jugs outside the error circles. The term is calculated from the binomial distribution (see Equation A3). The second term is the probability of adjacent jugs being on the left and right sides of their respective error circles (Figure 4). The theory of runs can replace the diagrams, to a large extent, for calculating the second term.

$$P_i(E_2,E_1 \mid E_0) = \sum_{\text{all possible runs}} P(E_2,E_1 \mid \text{run})P(\text{run} \mid i \text{ outside error circle}). \tag{B2}$$

The compound event E_2,E_1 is failure to detonate the whole line of clouds, i.e., "partial failure".

A run is an unbroken sequence, which in this application refers to a jug placed outside the error circle of the crane (0). A jug being on the ideal mark (+) starts a run of another kind. The runs for the +'s do not concern us. Consider a 3-jug line and the sequences that can occur with one jug out.

+ + 0   + 0 +   0 + +

There are three runs of length one for the 0's and they are designated 1_3,

i.e., (run $\text{length}_{\text{number of runs}}$), *designated by* r_k. (B3)

With two jugs out, the sequences are drawn as:

+ 0 0   0 + 0   0 0 +

Obviously the probability of a run of length two is 2/3. For even low numbers of jugs (see Appendix A), it is difficult to picture all the ways the jugs could be misplaced. The theory calculates the probability of a particular run arising. From Reference B1, the probability of having exactly k runs of 0's amounting to r_k in all is,

$$P(r_k) = \binom{r_1 - 1}{k - 1}\binom{r_2 + 1}{k} \Bigg/ \binom{r_1 + r_2}{r_1} \tag{B4}$$

and the probability of the 0's being arranged in v multiple runs is,

$$P(r_{k_1}, r_{k_2}, \ldots, r_{k_v}) = \frac{k!}{k_1!k_2!\ldots k_v!}\binom{r_2 + 1}{k} \Bigg/ \binom{r_1 + r_2}{r_1} \tag{B5}$$

where r_1 is the number of 0's and r_2 is the number of +'s; $k_1 + k_2 + \ldots + k_v = k$.

B1 *Feller, William*, An Introduction to Probability Theory and Its Applications, Vol I, Third Edition, John Wiley & Sons, Inc., p. 62.

The notation used in Equations B4 and B5 is the familiar,

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}\text{, where r n are any positive integers.} \qquad \text{(B6)}$$

For the 3-jug line with two jugs out of place there are, from the drawing after Equation B3, only two possible sequences: one run of length two or two runs of length one. So for run 2_1: $r_1 = 2$, $k = 1$; $r_2 = 3 - r_1 = 1$

1_2: $r_1 = 2$, $k = 2$; $r_2 = 3 - r_1 = 1$

From Equations B3 and B4 the probability of the single run arising is,

$$P(2_1) = \binom{2-1}{1-1}\binom{1+1}{1}\Big/\binom{2+1}{2} = \binom{1}{0}\binom{2}{1}\Big/\binom{3}{2} = 2/3 ,$$

(For a 3-jug line the run 2_1 looks like: + 0 0 or 0 0 +)

$$P(1_2) = \binom{2-1}{2-1}\binom{1+1}{2}\Big/\binom{2+1}{2} = 1/3 . \qquad \text{(B7)}$$

(For a 3-jug line the run 1_2 looks like: 0 + 0)

Equation B5 also gives $P(1_2)$ when $k = 2$, $k_1 = 2$, $k_2 = k_3 = 0$.

Note that the run probabilities agree with the probabilities from the explicit pictures in Appendix A and also that the probabilities sum to unity.

The first term in Equation B1 can be determined by the condition for failure, i.e., a left-right jug placement occurs, by using common sense or the pictures in Appendix A. For example, the run 1_2 cannot cause failure since there are not two adjacent jugs. If the run 2_1 occurs (the probability is 2/3 that it will, if the crane mislays two jugs out of three) failure is possible. The probability of failure is 1/4 when the run does occur because of the simple arrangement <u>+ 0 0</u> or for the <u>0 0 +</u> arrangement.

```
+   L   L
+   L   R * (failure)
+   R   L
+   R   R  1/4
```

The 3-jug example will now be written out. In place of the event notation, we write: f = failure = E_1, E_2, so Equation B1 restated for a 3-jug line is:

<u>Three-jug line:</u>

$$P(\text{failure}) = P(0\ \text{out})P(f|0\ \text{out}) + P(1\ \text{out})P(f|1\ \text{out})$$

$$+ P(2\ \text{out})P(f|2\ \text{out}) + P(3\ \text{out})P(f|3\ \text{out}). \tag{B8}$$

The conditional probabilities require Equation B2 to be restated,

$$P(f|0\ \text{out}) = 0$$

$$P(f|1\ \text{out}) = P(1_1)P(f|1_1) = (1)(0) = 0$$

$$P(f|2\ \text{out}) = P(1_2)P(f|1_2) + P(2_1)P(f|2_1) = (1/3)(0) + (2/3)(1/4) = 2/12$$

$$P(f|3\ \text{out}) = P(3_1)P(f|3_1) = (1)(1/2) = 1/2$$

Recall from Appendix A, the 3-jug line, Equation A6,

$$P(0\ \text{out}) = C_0^3 p^0(1 - p)^{3-0} = C_0^3(0.135)^0(0.865)^3 = 0.6472, \text{ etc.}$$

Now Equation B8 can be evaluated,

$$P(\text{failure}) = (0.6472)(0) + 0.3030(0) + 0.0473(2/12) + 0.00246(1/2)$$

P(failure) = 0.00911 for three jugs, in agreement with Appendix A and Table 1.

<u>Ten-jug line:</u>

Equation B1 with failure $(f) = (E_2,E_1)$ for ten jugs is written:

$$P(f) = P(0\ \text{out})P(f|0\ \text{out}) + P(1\ \text{out})P(f|1\ \text{out}) + \ldots + P(10\ \text{out})P(f|10\ \text{out}). \tag{B9}$$

Since events (E_2,E_1) cannot occur with zero or one jug mislaid,

$P(f|0\ \text{out}) = P(f|1\ \text{out}) = 0$. A further simplification by the binomial distribution (Equation A3) is that P(i out) < 10E-3 for i > 6, i.e., there is not much chance that the crane will mislay six jugs. The calculation of Equation B9 contains a product involving $p^6 = (0.135)E6 \simeq 10E-5$, so the equation can be simplified to a much shorter summation.

$$P(f) = P(2)P(f|2\ \text{out}) + P(3)P(f|3\ \text{out}) + P(4)P(f|4\ \text{out}) + P(5)P(f|5\ \text{out}) + e, \tag{B10}$$

where e < 10E-3.

The conditional probabilities are, by Equation B2, all combinations of runs which would place i jugs outside the crane error circle.

$$P(f|2\ \text{out}) = P(1_2)P(f|1_2) + P(2_1)P(f|2_1) = (36/45)(0) + (9/45)(1/4) = 0.0500,$$

by Equation B4 and the simple diagram of probability of failure for two jugs.

$$P(f|3\ \text{out}) = P(1_3)P(f|1_3) + P(1_1,2_1)P(f|1_1,2_1) + P(3_1)P(f|3_1) \tag{B11}$$

$$= (56/120)(0) + (56/120)(1/4) + (8/120)(1/2) = 0.150.$$

Note that a multiple run occurred in Equation B11 so Equation B5 is needed.

$$P(1_1,2_1) = \frac{2!}{1!1!}\binom{7+1}{2}\Big/\binom{3+7}{3} = 56/120. \qquad (B12)$$

The probability of failure for the multiple run $(1_1,2_1)$ is argued as follows. For success, the detonation wave must travel through one of the runs, say 1_1, then travel through the next run 2_1. Since the two events are independent of each other, the probability of doing both is the product $P(1_1)P(2_1)|$success; and the probability of not traveling the 10-jug line holding these particular runs is the complement.

$$P(f|1_1,2_1) = 1 - P(1_1)P(2_1)|\text{success} = 1 - (1)(3/4) = 1/4.$$

$$P(f|4\ \text{out}) = P(1_4)P(f|1_4) + P(4_1)P(f|4_1) + P(1_1,3_1)P(f|1_1,3_1) \qquad (B13)$$

$$+ P(2_2)P(f|2_2) + P(1_2,2_1)P(f|1_2,2_1)$$

$$= (35/210)(0) + (7/210)(11/16) + (42/210)[1 - (1)(4/8)]$$
$$+ (21/210)[1 - (3/4)**2] + (105/210)(1/4)$$

$$= 0.2917.$$

$$P(f|5\ \text{out}) = P(1_5)P(f|1_5) + P(1_3,2_1)P(f|1_3,2_1) + P(1_2,3_1)P(f|1_2,3_1)$$

$$+ P(1_1,2_2)P(f|1_1,2_2) + P(2_1,3_1)P(f|2_1,3_1)$$

$$+ P(1_1,4_1)P(f|1_1,4_1) + P(5_1)P(f|5_1)$$

$$= (6/252)(0) + (60/252)(1/4) + (60/252)(1/2)$$
$$+ (60/252)[1 - (3/4)**2] + (30/252)[1 - 1(11/16)]$$
$$+ (30/252)[1 - (3/4)(1/2)] + (6/252)(26/32)$$

$$= 0.4583.$$

Note that the runs 4_1 and 5_1 use the diagrams of Appendix A to supply the conditional probabilities $P(f|4)$ and $P(f|5)$. Under the assumption that runs of six or higher need not be evaluated (because their contribution to the probability of failure tends to zero), no more diagrams need be prepared. Note for checking purposes that the run probabilities of i jugs being outside the crane error circle sum to one. The terms P(2), etc., in Equation B10 are found as usual by the binomial distribution,

$$P(2\ \text{out}) = C^{10}_{2}\, p^{2}(1-p)^{10-2} = C^{10}_{2}(0.135)^{2}(0.865)^{8} = 0.2570.$$

So Equation B10 becomes,

$$P(\text{failure}) \simeq (.2570)(.05) + (.1070)(.15) + (.02922)(.2917) + (.00547)(.4583)$$

$$\simeq 0.0399 \text{ for ten jugs, against the correct value of } 0.0404 \text{ in Table 1.} \qquad (B14)$$

The reduction in terms made in Equation B10 is argued as follows. The probability of the crane mislaying six jugs tends to zero, e.g., P(6 out) = 0.000, so the terms pertaining to larger jug numbers go to zero and the sum of those higher terms does not contribute to P(failure). In fact, the argument is false as computer calculations show. The practical effect of accepting the argument is to mislead one into thinking that long juglines are no more apt to fail than a 6-jug line. Actually, probability of failure is almost linear with jug numbers up to 30, as shown in Figure 6. That figure also illustrates that the rate of increase of probability of failure is greater when $p = 0.2$ than when $p = 0.1$. When $p = 0.1$, the 30-jug line is not much more apt to fail than a 6-jug line (7% vs. 1%); however, when $p = 0.2$, the probability of failure goes from 5% for six jugs to 25% for 30 jugs. The upper slope ($p = 0.2$) is 3.3 times steeper than the lower slope ($p = 0.1$).

APPENDIX C

RUNS OF ARRANGEMENTS

In Appendix B the probability of failure was found by considering every possible arrangement of jugs. Here the probability of failure is found by considering all the possible arrangements of runs (one jug or consecutive jugs out of place), and the number of ways each of these arrangements can occur. A probability is calculated for each possible arrangement and summed over all arrangements to give the overall probability of failure.

The equations of this method are further explained. The crane lays each jug in or out of the allowable error circle around the ideal mark. Like items that individually have a two-state existence (in or out, up or down, heads or tails) and are placed or counted together have a probability of a number of each state arising. This is named a Bernoulli or binomial process. By that process the probability P of laying n_{out} jugs out and the probability Q of laying the remaining $(n - n_{out})$ jugs in is the product of each independent laying event,

$$P(\text{arrangement}) = P^{n_{out}} Q^{n-n_{out}}, \quad \text{where } Q = (1 - P). \tag{C1}$$

This equation only recognizes the count of so many jugs out and the complementary number in. It does not recognize the order of the outs and ins (permutations or arrangements of jugs).

Several different arrangements of n jugs having the same number n_{out} out are possible. Call the number of such arrangements n_L. All of these arrangements were pictured in Appendix A for small jug lines, i.e., n = 3, 4, 5. Drawing more was cumbersome. The following formula was found which gives the number of arrangements possible,

$$n_L = \binom{n - n_{out} + 1}{R} \tag{C2}$$

where R = number of runs in the arrangement of jugs. Because each arrangement has the same probability of arising, we multiply Equations C1 and C2 to get the probability of all arrangements of R runs with n_{out} jugs mislaid.

$$P(\text{arrangement}) = \binom{n - n_{out} + 1}{R} P^{n_{out}} (1 - P)^{n-n_{out}} \tag{C3}$$

Only some arrangements of jugs will cause partial failure to detonate the cloud line. The bad arrangements (Figure 4) are ones with adjacent jugs on the left and right side of the crane error circle. Testing and comparison with the diagrams of Appendix A show that the portion of arrangements, i.e., probability of failure, which will give partial failure is

$$1 - [(L + 1)/2^{**}L] \tag{C4}$$

for one run of length L or

$$1 - [1 - P(L_1)][1 - P(L_2)][1 - P(L_3)] \ldots \tag{C5}$$

for multiple runs where $P(L_1)$, $P(L_2)$ are given by Equation C3. The reasoning behind Equation C4 was given beside Equation B11.

The probability of partial failure is the probability of the dependent joint events of: (1) an arrangement arising and (2) the probability of failure with that arrangement. This statement is written as

$$P(\text{failure with an arrangement}) = P(f,A) = P(f|A)\ P(A), \tag{C6}$$

where A is any arrangement of jugs in or out of the error circle.

The total probability of failure is the sum of the independent arrangements' probabilities of failure,

$$P(\text{failure}) = \sum P(\text{failure with an arrangement}). \tag{C7}$$

For comparison with the method of Appendix B, Equation B7, the 3-jug line problem is worked again in Table C1. For comparison with the method of Appendix A, Equations A9-A11, a 5-jug line is worked again in Table C2. These calculations use the probability of mislaying one jug as p = 0.135.

The method of this appendix was coded in FORTRAN 77 and the program, named JUGRUN, is listed in Table C3. The results tables were generated by the program. The program is efficient up to n = 25 jugs but gets very time consuming to run after that. Some shorter, faster programs based on set theory (not run theory) will be listed in Appendix D.

TABLE C1. Probability of Failure, Runs of Arrangements, 3-Jug Line.

arrangements of one run of length one

R = 1	+	+	0
L = 1	+	0	+
	0	+	+

arrangements of 1 or 2 runs of length 2 or 1

R = 1	+	0	0
L = 2	0	0	+
R = 2	0	+	0
L = 1			

arrangement of one run of length three

R = 1	0	0	0
L = 3			

R no. of runs	L length of runs	P(f¦L) (Note 1)	n_L	P(L) (Note 2)	P(failure)
	$n_{out} = L$	$1 - \frac{(L+1)}{2^L}$	$\binom{n-n_{out}+1}{R}$		$n_L P(f\mid L)P(L)$
1	1	0	3	$0.135^1\ 0.865^2 = 0.1010$	0
1	2	1/4	2	$0.135^2\ 0.865^1 = 0.0158$	0.0079
1	3	1/2	1	$0.135^3\ 0.865^0 = 0.0025$	0.0012
2	1,1 multiple run	1-(1-0)(1-0)	1	$0.135^2\ 0.865^1 = 0.0158$	0
					0.0091

The probability of failure for the 3-jug line is 0.0091 as shown in Table 1 and earlier appendices.

Note 1. Probability of failure given this run length.
Note 2. Probability of getting this many out and in. Binomial formula:

$$P(L) = P^{n_{out}}\ (1-P)^{n-n_{out}}$$

TABLE C2. Probability of Failure, Runs of Arrangements, 5-Jug Line.

R no. of runs	L length of runs	P(f|L) (Note 1)	n_L	P(L) (Note 2)	P(failure)
	$n_{out} = L$	$1 - \frac{(L+1)}{2^L}$	$\binom{n-n_{out}+1}{R}$		$n_L P(f\|L)P(L)$
1	1	0	5	$0.135^1\ 0.865^4 = 0.075578$	0
1	2	1/4	4	$0.135^2\ 0.865^3 = 0.011795$	0.011795
1	3	1/2	3	$0.135^3\ 0.865^2 = 0.001841$	0.002762
1	4	11/16	2	$0.135^4\ 0.865^1 = 0.000287$	0.000395
1	5	26/32	1	$0.135^5\ 0.865^0 = 0.000045$	0.000037
2	1,1	1-(1-0)(1-0)	6	$0.135^2\ 0.865^3 = 0.011795$	0
2	1,2	1-(1-0)(1-1/4)	3	$0.135^3\ 0.865^2 = 0.001841$	0.001381
2	1,3	1-(1-1/4)(1-1/2)	1	$0.135^4\ 0.865^1 = 0.000287$	0.000144
2	2,1	1-(1-1/4)(1-1/2)	3	$0.135^3\ 0.865^2 = 0.001841$	0.001381
2	2,2	1-(1-1/4)(1-1/4)	1	$0.135^4\ 0.865^1 = 0.000287$	0.000126
2	3,1	1-(1-1/2)(1-0)	1	$0.135^4\ 0.865^1 = 0.000287$	0.000144
3	1,1,1	1-(1-0)(1-0)(1-0)	1	$0.135^3\ 0.865^2 = 0.001841$	0
					0.018165
				shown in Table 1:	0.0182

Note 1. Probability of failure given this run length.

Note 2. Probability of getting this many out and in. Binomial formula:

$$P(L) = P^{n_{out}} (1-P)^{n-n_{out}}$$

TABLE C3. JUGRUN Program.

```
      PROGRAM JUGRUN1
C     PROGRAM TO CALCULATE THE PROBABILITY OF FAILURE TO
C     DETONATE DUE TO CRANE ERROR (MISPLACED JUGS)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION CPROB(30),PROB(30)
      WRITE(6,1)
    1 FORMAT(//'CALCULATION OF PROBABILITY OF FAILURE TO DETONATE'/
     *'DUE TO CRANE ERROR (MISPLACED JUGS)'//
     *'WHEN PROMPTED, ENTER N, THE NUMBER OF JUGS IN A ROW,'/
     *'AND P, THE PROBABILITY OF AN INDIVIDUAL JUG BEING LAID'/
     *'OUTSIDE ITS ERROR CIRCLE.'//
     *'N MUST BE A POSITIVE INTEGER NOT GREATER THAN 30,'/
     *'AND P MUST BE GREATER THAN 0 AND LESS THAN 1.'//
     *'ENTER ZEROES FOR N AND P WHEN NO MORE DATA ARE TO BE ENTERED.'/)
    2 READ(5,*)N,P
      IF(N.EQ.0)GO TO 21
C     COMPUTE CONDITIONAL PROBABILITY OF FAILURE GIVEN
C     I CONSECUTIVE JUGS ARE OUTSIDE THEIR ERROR CIRCLES,
C     AND PROBABILITY OF I JUGS BEING PLACED OUTSIDE
C     THEIR ERROR CIRCLES, FOR I = 1, 2, ..., N
      Q=1.0-P
      DO 3 I=1,N
      CPROB(I)=1.0-(I+1.0)/2**I
    3 PROB(I)=P**I*Q**(N-I)
      PRFAIL=0.0
C     CALCULATIONS FOR 1 RUN
      DO 4 NOUT=1,N
      NFREQ=NCOMB(N+1-NOUT,1)
      LRUN1=NOUT
      PROBC=CPROB(LRUN1)
    4 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
C     CALCULATIONS FOR 2 RUNS (MUST HAVE AT LEAST 3 JUGS)
      IF(N.LT.3)GO TO 19
      DO 5 NOUT=2,N-1
      NFREQ=NCOMB(N+1-NOUT,2)
      DO 5 LRUN1=1,NOUT-1
      LRUN2=NOUT-LRUN1
      PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))
    5 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
C     CALCULATIONS FOR 3 RUNS (MUST HAVE AT LEAST 5 JUGS)
      IF(N.LT.5)GO TO 19
      DO 6 NOUT=3,N-2
      NFREQ=NCOMB(N+1-NOUT,3)
      DO 6 LRUN1=1,NOUT-2
      DO 6 LRUN2=1,NOUT-1-LRUN1
      LRUN3=NOUT-LRUN1-LRUN2
      PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
    6 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
```

TABLE C3. (continued)

```
C     CALCULATIONS FOR 4 RUNS (MUST HAVE AT LEAST 7 JUGS)
        IF(N.LT.7)GO TO 19
        DO 7 NOUT=4,N-3
        NFREQ=NCOMB(N+1-NOUT,4)
        DO 7 LRUN1=1,NOUT-3
        DO 7 LRUN2=1,NOUT-2-LRUN1
        DO 7 LRUN3=1,NOUT-1-LRUN1-LRUN2
        LRUN4=NOUT-LRUN1-LRUN2-LRUN3
        PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
       **(1.0-CPROB(LRUN4))
      7 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
C     CALCULATIONS FOR 5 RUNS (MUST HAVE AT LEAST 9 JUGS)
        IF(N.LT.9)GO TO 19
        DO 8 NOUT=5,N-4
        NFREQ=NCOMB(N+1-NOUT,5)
        DO 8 LRUN1=1,NOUT-4
        DO 8 LRUN2=1,NOUT-3-LRUN1
        DO 8 LRUN3=1,NOUT-2-LRUN1-LRUN2
        DO 8 LRUN4=1,NOUT-1-LRUN1-LRUN2-LRUN3
        LRUN5=NOUT-LRUN1-LRUN2-LRUN3-LRUN4
        PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
       **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))
      8 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
C     CALCULATIONS FOR 6 RUNS (MUST HAVE AT LEAST 11 JUGS)
        IF(N.LT.11)GO TO 19
        DO 9 NOUT=6,N-5
        NFREQ=NCOMB(N+1-NOUT,6)
        DO 9 LRUN1=1,NOUT-5
        DO 9 LRUN2=1,NOUT-4-LRUN1
        DO 9 LRUN3=1,NOUT-3-LRUN1-LRUN2
        DO 9 LRUN4=1,NOUT-2-LRUN1-LRUN2-LRUN3
        DO 9 LRUN5=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4
        LRUN6=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
        PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
       **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))
      9 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
C     CALCULATIONS FOR 7 RUNS (MUST HAVE AT LEAST 13 JUGS)
        IF(N.LT.13)GO TO 19
        DO 10 NOUT=7,N-6
        NFREQ=NCOMB(N+1-NOUT,7)
        DO 10 LRUN1=1,NOUT-6
        DO 10 LRUN2=1,NOUT-5-LRUN1
        DO 10 LRUN3=1,NOUT-4-LRUN1-LRUN2
        DO 10 LRUN4=1,NOUT-3-LRUN1-LRUN2-LRUN3
        DO 10 LRUN5=1,NOUT-2-LRUN1-LRUN2-LRUN3-LRUN4
        DO 10 LRUN6=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
        LRUN7=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6
        PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
       **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))*
       *(1.0-CPROB(LRUN7))
     10 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
```

TABLE C3. (continued)

```
C    CALCULATIONS FOR 8 RUNS (MUST HAVE AT LEAST 15 JUGS)
      IF(N.LT.15)GO TO 19
      DO 11 NOUT=8,N-7
      NFREQ=NCOMB(N+1-NOUT,8)
      DO 11 LRUN1=1,NOUT-7
      DO 11 LRUN2=1,NOUT-6-LRUN1
      DO 11 LRUN3=1,NOUT-5-LRUN1-LRUN2
      DO 11 LRUN4=1,NOUT-4-LRUN1-LRUN2-LRUN3
      DO 11 LRUN5=1,NOUT-3-LRUN1-LRUN2-LRUN3-LRUN4
      DO 11 LRUN6=1,NOUT-2-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
      DO 11 LRUN7=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6
      LRUN8=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7
      PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
     **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))*
     *(1.0-CPROB(LRUN7))*(1.0-CPROB(LRUN8))
   11 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
C    CALCULATIONS FOR 9 RUNS (MUST HAVE AT LEAST 17 JUGS)
      IF(N.LT.17)GO TO 19
      DO 12 NOUT=9,N-8
      NFREQ=NCOMB(N+1-NOUT,9)
      DO 12 LRUN1=1,NOUT-8
      DO 12 LRUN2=1,NOUT-7-LRUN1
      DO 12 LRUN3=1,NOUT-6-LRUN1-LRUN2
      DO 12 LRUN4=1,NOUT-5-LRUN1-LRUN2-LRUN3
      DO 12 LRUN5=1,NOUT-4-LRUN1-LRUN2-LRUN3-LRUN4
      DO 12 LRUN6=1,NOUT-3-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
      DO 12 LRUN7=1,NOUT-2-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6
      DO 12 LRUN8=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7
      LRUN9=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-LRUN8
      PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
     **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))*
     *(1.0-CPROB(LRUN7))*(1.0-CPROB(LRUN8))*(1.0-CPROB(LRUN9))
   12 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
C    CALCULATIONS FOR 10 RUNS (MUST HAVE AT LEAST 19 JUGS)
      IF(N.LT.19)GO TO 19
      DO 13 NOUT=10,N-9
      NFREQ=NCOMB(N+1-NOUT,10)
      DO 13 LRUN1=1,NOUT-9
      DO 13 LRUN2=1,NOUT-8-LRUN1
      DO 13 LRUN3=1,NOUT-7-LRUN1-LRUN2
      DO 13 LRUN4=1,NOUT-6-LRUN1-LRUN2-LRUN3
      DO 13 LRUN5=1,NOUT-5-LRUN1-LRUN2-LRUN3-LRUN4
      DO 13 LRUN6=1,NOUT-4-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
      DO 13 LRUN7=1,NOUT-3-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6
      DO 13 LRUN8=1,NOUT-2-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7
      DO 13 LRUN9=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8
      LRUN10=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-LRUN8-LRUN9
      PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
     **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))*
     *(1.0-CPROB(LRUN7))*(1.0-CPROB(LRUN8))*(1.0-CPROB(LRUN9))*
     *(1.0-CPROB(LRUN10))
   13 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
```

TABLE C3. (continued)

```
C   CALCULATIONS FOR 11 RUNS (MUST HAVE AT LEAST 21 JUGS)
      IF(N.LT.21)GO TO 19
      DO 14 NOUT=11,N-10
      NFREQ=NCOMB(N+1-NOUT,11)
      DO 14 LRUN1=1,NOUT-10
      DO 14 LRUN2=1,NOUT-9-LRUN1
      DO 14 LRUN3=1,NOUT-8-LRUN1-LRUN2
      DO 14 LRUN4=1,NOUT-7-LRUN1-LRUN2-LRUN3
      DO 14 LRUN5=1,NOUT-6-LRUN1-LRUN2-LRUN3-LRUN4
      DO 14 LRUN6=1,NOUT-5-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
      DO 14 LRUN7=1,NOUT-4-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6
      DO 14 LRUN8=1,NOUT-3-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7
      DO 14 LRUN9=1,NOUT-2-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8
      DO 14 LRUN10=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8-LRUN9
      LRUN11=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-LRUN8-LRUN9-
     *LRUN10
      PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
     **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))*
     *(1.0-CPROB(LRUN7))*(1.0-CPROB(LRUN8))*(1.0-CPROB(LRUN9))*
     *(1.0-CPROB(LRUN10))*(1.0-CPROB(LRUN11))
   14 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
C   CALCULATIONS FOR 12 RUNS (MUST HAVE AT LEAST 23 JUGS)
      IF(N.LT.23)GO TO 19
      DO 15 NOUT=12,N-11
      NFREQ=NCOMB(N+1-NOUT,12)
      DO 15 LRUN1=1,NOUT-11
      DO 15 LRUN2=1,NOUT-10-LRUN1
      DO 15 LRUN3=1,NOUT-9-LRUN1-LRUN2
      DO 15 LRUN4=1,NOUT-8-LRUN1-LRUN2-LRUN3
      DO 15 LRUN5=1,NOUT-7-LRUN1-LRUN2-LRUN3-LRUN4
      DO 15 LRUN6=1,NOUT-6-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
      DO 15 LRUN7=1,NOUT-5-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6
      DO 15 LRUN8=1,NOUT-4-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7
      DO 15 LRUN9=1,NOUT-3-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8
      DO 15 LRUN10=1,NOUT-2-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8-LRUN9
      DO 15 LRUN11=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8-LRUN9-LRUN10
      LRUN12=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-LRUN8-LRUN9-
     *LRUN10-LRUN11
      PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
     **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))*
     *(1.0-CPROB(LRUN7))*(1.0-CPROB(LRUN8))*(1.0-CPROB(LRUN9))*
     *(1.0-CPROB(LRUN10))*(1.0-CPROB(LRUN11))*(1.0-CPROB(LRUN12))
   15 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
```

TABLE C3. (continued)

```
C     CALCULATIONS FOR 13 RUNS (MUST HAVE AT LEAST 25 JUGS)
        IF(N.LT.25)GO TO 17
        DO 16 NOUT=13,N-12
        NFREQ=NCOMB(N+1-NOUT,13)
        DO 16 LRUN1=1,NOUT-12
        DO 16 LRUN2=1,NOUT-11-LRUN1
        DO 16 LRUN3=1,NOUT-10-LRUN1-LRUN2
        DO 16 LRUN4=1,NOUT-9-LRUN1-LRUN2-LRUN3
        DO 16 LRUN5=1,NOUT-8-LRUN1-LRUN2-LRUN3-LRUN4
        DO 16 LRUN6=1,NOUT-7-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
        DO 16 LRUN7=1,NOUT-6-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6
        DO 16 LRUN8=1,NOUT-5-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7
        DO 16 LRUN9=1,NOUT-4-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
       *LRUN8
        DO 16 LRUN10=1,NOUT-3-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
       *LRUN8-LRUN9
        DO 16 LRUN11=1,NOUT-2-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
       *LRUN8-LRUN9-LRUN10
        DO 16 LRUN12=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
       *LRUN8-LRUN9-LRUN10-LRUN11
        LRUN13=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-LRUN8-LRUN9-
       *LRUN10-LRUN11-LRUN12
        PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
       **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))*
       *(1.0-CPROB(LRUN7))*(1.0-CPROB(LRUN8))*(1.0-CPROB(LRUN9))*
       *(1.0-CPROB(LRUN10))*(1.0-CPROB(LRUN11))*(1.0-CPROB(LRUN12))*
       *(1.0-CPROB(LRUN13))
     16 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
C     CALCULATIONS FOR 14 RUNS (MUST HAVE AT LEAST 27 JUGS)
        IF(N.LT.27)GO TO 19
        DO 17 NOUT=14,N-13
        NFREQ=NCOMB(N+1-NOUT,14)
        DO 17 LRUN1=1,NOUT-13
        DO 17 LRUN2=1,NOUT-12-LRUN1
        DO 17 LRUN3=1,NOUT-11-LRUN1-LRUN2
        DO 17 LRUN4=1,NOUT-10-LRUN1-LRUN2-LRUN3
        DO 17 LRUN5=1,NOUT-9-LRUN1-LRUN2-LRUN3-LRUN4
        DO 17 LRUN6=1,NOUT-8-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
        DO 17 LRUN7=1,NOUT-7-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6
        DO 17 LRUN8=1,NOUT-6-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7
        DO 17 LRUN9=1,NOUT-5-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
       *LRUN8
        DO 17 LRUN10=1,NOUT-4-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
       *LRUN8-LRUN9
        DO 17 LRUN11=1,NOUT-3-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
       *LRUN8-LRUN9-LRUN10
        DO 17 LRUN12=1,NOUT-2-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
       *LRUN8-LRUN9-LRUN10-LRUN11
        DO 17 LRUN13=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
       *LRUN8-LRUN9-LRUN10-LRUN11-LRUN12
        LRUN14=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-LRUN8-LRUN9-
       *LRUN10-LRUN11-LRUN12-LRUN13
        PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
       **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))*
       *(1.0-CPROB(LRUN7))*(1.0-CPROB(LRUN8))*(1.0-CPROB(LRUN9))*
       *(1.0-CPROB(LRUN10))*(1.0-CPROB(LRUN11))*(1.0-CPROB(LRUN12))*
       *(1.0-CPROB(LRUN13))*(1.0-CPROB(LRUN14))
     17 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
```

TABLE C3. (continued)

```
C     CALCULATIONS FOR 15 RUNS (MUST HAVE AT LEAST 29 JUGS)
      IF(N.LT.29)GO TO 19
      DO 18 NOUT=15,N-14
      NFREQ=NCOMB(N+1-NOUT,15)
      DO 18 LRUN1=1,NOUT-14
      DO 18 LRUN2=1,NOUT-13-LRUN1
      DO 18 LRUN3=1,NOUT-12-LRUN1-LRUN2
      DO 18 LRUN4=1,NOUT-11-LRUN1-LRUN2-LRUN3
      DO 18 LRUN5=1,NOUT-10-LRUN1-LRUN2-LRUN3-LRUN4
      DO 18 LRUN6=1,NOUT-9-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5
      DO 18 LRUN7=1,NOUT-8-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6
      DO 18 LRUN8=1,NOUT-7-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7
      DO 18 LRUN9=1,NOUT-6-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8
      DO 18 LRUN10=1,NOUT-5-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8-LRUN9
      DO 18 LRUN11=1,NOUT-4-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8-LRUN9-LRUN10
      DO 18 LRUN12=1,NOUT-3-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8-LRUN9-LRUN10-LRUN11
      DO 18 LRUN13=1,NOUT-2-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8-LRUN9-LRUN10-LRUN11-LRUN12
      DO 18 LRUN14=1,NOUT-1-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-
     *LRUN8-LRUN9-LRUN10-LRUN11-LRUN12-LRUN13
      LRUN15=NOUT-LRUN1-LRUN2-LRUN3-LRUN4-LRUN5-LRUN6-LRUN7-LRUN8-LRUN9-
     *LRUN10-LRUN11-LRUN12-LRUN13-LRUN14
      PROBC=1.0-(1.0-CPROB(LRUN1))*(1.0-CPROB(LRUN2))*(1.0-CPROB(LRUN3))
     **(1.0-CPROB(LRUN4))*(1.0-CPROB(LRUN5))*(1.0-CPROB(LRUN6))*
     *(1.0-CPROB(LRUN7))*(1.0-CPROB(LRUN8))*(1.0-CPROB(LRUN9))*
     *(1.0-CPROB(LRUN10))*(1.0-CPROB(LRUN11))*(1.0-CPROB(LRUN12))*
     *(1.0-CPROB(LRUN13))*(1.0-CPROB(LRUN14))*(1.0-CPROB(LRUN15))
   18 PRFAIL=PRFAIL+NFREQ*PROBC*PROB(NOUT)
   19 WRITE(6,20)N,P,PRFAIL
   20 FORMAT(/'N =',T18,I2/
     *'P =',T14,F6.4//
     *'PROBABILITY'/
     *'OF FAILURE = ',F6.4/)
      GO TO 2
   21 STOP
      END
      FUNCTION NCOMB(N,M)
      M1=N-M
      IF(M.LE.M1)THEN
      MM=M
      ELSE
      MM=M1
      END IF
      NCOMB=1
      IF(MM.NE.0)THEN
      DO 1 I=1,MM
    1 NCOMB=NCOMB*(N+1-I)/I
      END IF
      END
```

APPENDIX D

THE BAD EVENT

The JUGFAE probability of failure due to improper crane placement can be computed without using conditional probability or run theory. Two easily programmable methods for doing exact calculations are given as well as an approximate method which would be useful for field work when computing facilities are not available.

The first method, discussed in Section D.1, runs into some numerical stability problems for very large n and large p (close to 1) but these cases are operationally unimportant. For the value of p (probability of misplacing one jug) that is considered in this work ($p = 0.135$) the method works for any n. The program listing of this method, called JUGCOM, is in Table D1.

The second method avoids the stability problem, although for a given p all failure probabilities for 1, 2, ..., n-1 must be computed in order to arrive at the n-th probability. This recursion is only a minor problem in terms of computer time. By comparison, the run theory JUGRUN program is only valid for $n < 31$ and takes much more time. The program listing of this method, called JUGREC, is in Table D2.

D.1 Exact calculation, JUGCOM. We have a failure if one jug is placed too far to the left and the next one (to the right of the first one) is placed too far to the right. Thus, we have a failure in a line of n jugs if the following occurs:

(jug 1 too far left and jug 2 too far right) or
(jug 2 too far left and jug 3 too far right) or
⋮
(jug n-2 too far left and jug n-1 too far right) or
(jug n-1 too far left and jug n too far right).

The placement of each jug is assumed to be independent of the placement of the other jugs.

The probability of failure for an n-jug line may be written as

$$P(\text{failure}) = P(E_1 \cup E_2 \cup \ldots \cup E_{n-2} \cup E_{n-1}), \tag{D1}$$

where E_i = the event: jug i too far left and jug i+1 too far right, for i = 1, 2, ..., n-1 and "$\cup$" means "or".
Note that events E_1 and E_2 are defined quite differently in Appendix A.

Equation 3 indicated the expansion of Equation 2 for two sets; for n sets,

$$P(E_1 \cup E_2 \cup \ldots \cup E_{n-2} \cup E_{n-1}) = \sum_{i=1}^{n-1} P(E_i) + \sum_{j=1}^{n-2} (-1)^j(\text{j-way intersections}), \tag{D2}$$

$$P(E_1 \cup E_2 \cup \cdots \cup E_{n-2} \cup E_{n-1}) = \text{individual probabilities}$$
$$- \text{1-way intersections}$$
$$+ \text{2-way intersections}$$
$$\vdots$$
$$+ (-1)^{n-2}[(n-2)\text{-way intersections}], \tag{D3}$$

$$= [P(E_1) + P(E_2) + \cdots + P(E_{n-2}) + P(E_{n-1})]$$
$$- [P(E_1 \cap E_2) - P(E_1 \cap E_3) - \cdots - P(E_{n-2} \cap E_{n-1})]$$
$$+ [P(E_1 \cap E_2 \cap E_3) + \cdots + P(E_{n-3} \cap E_{n-2} \cap E_{n-1})]$$
$$\vdots$$
$$+ (-1)^{n-2}[P(E_1 \cap E_2 \cap \cdots \cap E_{n-2} \cap E_{n-1})], \tag{D4}$$

where "$\cup$" means "or" and "$\cap$" means "and".

Not all the E_i's are independent events. In particular, E_i and E_{i+1} are mutually exclusive events because E_i and E_{i+1} cannot both happen. (Jug i+1 cannot be both too far to the right and too far to the left.) However, all other subsets of events are independent, e.g., E_i and E_{i+2} are independent, E_i and E_{i+3} are independent, etc. In general, all intersections of E_i and $E_{i\pm1}$ events will be zero and all other intersections will be nonzero. All cases can be generalized in an expression for probability of failure P(f),

$$P(f) = \sum_{i=1}^{\lfloor n/2 \rfloor} (-1)^{i+1} \binom{n-i}{i} p'^{\,i}, \tag{D5}$$

where

- n = number of jugs in a row,
- $\lfloor n/2 \rfloor$ = the integer part of n/2,
- p = probability of an individual jug being outside error circle,
- $p/2$ = probability of an individual jug being too far left
  = probability of an individual jug being too far right,
- p' = $(p/2)^2$ = probability of one jug being too far left and the next jug being too far right (by independence of placements of jugs)
  = probability of a particular pair of consecutive jugs causing failure.

For computing purposes, rename the quantity after the summation sign T_i,

$$P(f) = \sum_{i=1}^{\lfloor n/2 \rfloor} T_i . \tag{D6}$$

A recursive formula can be obtained by substituting i-1 for i in T_i and finding the ratio T_i/T_{i+1}. Letting $T_0 = -1$, we have

$$T_i = \frac{-(n-2i+1)(n-2i+2)}{i(n-i+1)} (p')(T_{i-1}), \quad i = 1, 2, \ldots, \lfloor n/2 \rfloor. \tag{D7}$$

Equations D6 and D7 form the basis of the program JUGCOM.

As an example, we will handle a 5-jug line by expanding both Equation D4 and Equation D5.

a. Five-jug line by Equation D4.

$$
\begin{aligned}
P(f) = {} & P(E_1) + P(E_2) + P(E_3) + P(E_4) \\
& - \cancelto{0}{P(E_1 \cap E_2)} - P(E_1 \cap E_3) - P(E_1 \cap E_4) - \cancelto{0}{P(E_2 \cap E_3)} - P(E_2 \cap E_4) - \cancelto{0}{P(E_3 \cap E_4)} \\
& + \cancelto{0}{P(E_1 \cap E_2 \cap E_3)} + \cancelto{0}{P(E_1 \cap E_2 \cap E_4)} + \cancelto{0}{P(E_1 \cap E_3 \cap E_4)} + \cancelto{0}{P(E_2 \cap E_3 \cap E_4)} \\
& - \cancelto{0}{P(E_1 \cap E_2 \cap E_3 \cap E_4)}.
\end{aligned}
\tag{D8}
$$

Since adjacent events E_i and $E_{i\pm1}$ are mutually exclusive, the intersections of such events are the null set and the probability of the null set is zero. Then Equation D8 becomes

$$P(f) = P(E_1) + P(E_2) + P(E_3) + P(E_4) - P(E_1 \; E_3) - P(E_1 \; E_4) - P(E_2 \; E_4), \tag{D9}$$

but $P(E_i) = p'$, and P(independent events) is the product of the individual probabilities, so Equation D9 becomes

$$P(f) = 4p' - 3p'^2 . \tag{D10}$$

b. Five-jug line by Equation D5.

$$P(f) = \sum_{i=1}^{\lfloor 5/2 \rfloor} (-1)^{i+1} \binom{5 - i}{i} p'^{i} , \tag{D11}$$

recognize that $\lfloor 5/2 \rfloor = 2$, i.e., retain the quotient.

$$P(f) = (-1)^{1+1} \binom{5 - 1}{1} p'^{1} + (-1)^{2+1} \binom{5 - 2}{2} p'^{2}, \tag{D12}$$

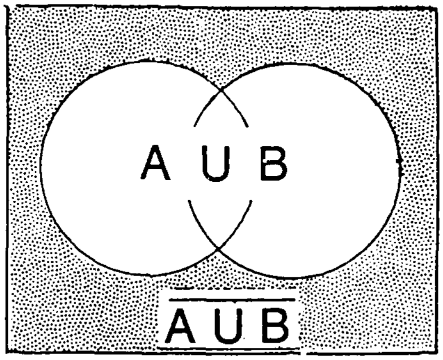
$$P(f) = 4p' - 3p'^2 . \tag{D10}$$

Both expansions reduce to the same equation for the probability of failure of a 5-jug line. Evaluating Equation D10 with p' = (p/2)**2 = (0.135/2)**2 = 0.00456, P(f) = 0.01816 ≃ 0.0182 as in Table 1, n = 5.

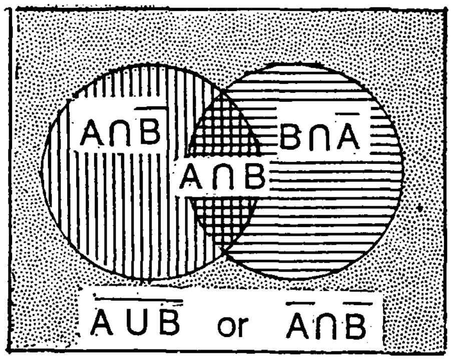
TABLE D1. JUGCOM Program.

```
      PROGRAM JUGCOM
C     PROGRAM TO CALCULATE THE PROBABILITY OF FAILURE TO
C     DETONATE DUE TO CRANE ERROR (MISPLACED JUGS)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      WRITE(6,1)
    1 FORMAT(/'CALCULATION OF PROBABILITY OF FAILURE TO DETONATE'/
     *'DUE TO CRANE ERROR (MISPLACED JUGS)'//
     *'WHEN PROMPTED, ENTER N, THE NUMBER OF JUGS IN A ROW,'/
     *'AND P, THE PROBABILITY OF AN INDIVIDUAL JUG BEING LAID'/
     *'OUTSIDE ITS ERROR CIRCLE.'//
     *'N MUST BE A POSITIVE INTEGER GREATER THAN OR EQUAL TO 2.'/
     *'AND P MUST BE GREATER THAN OR EQUAL TO 0 AND'/
     *'LESS THAN OR EQUAL TO 1.'//
     *'ENTER ZEROES FOR N AND P WHEN NO MORE DATA ARE TO BE ENTERED.'/)
    2 READ(5,*)N,P
      IF(N.EQ.0)GO TO 5
      P2=(P/2.0)**2
      NTERMS=N/2
      PTERM=-1.0
      PRFAIL=0.0
      DO 3 I=1,NTERMS
      PTERM=-PTERM*P2*((N+1.0-2.0*I)/(N+1.0-I))*((N+2.0-2.0*I)/I)
    3 PRFAIL=PRFAIL+PTERM
      WRITE(6,4)N,P,PRFAIL
    4 FORMAT(/'N =',T16,I4/
     *'P =',T14,F6.4//
     *'PROBABILITY'/
     *'OF FAILURE = ',F6.4/)
      GO TO 2
    5 STOP
      END
```

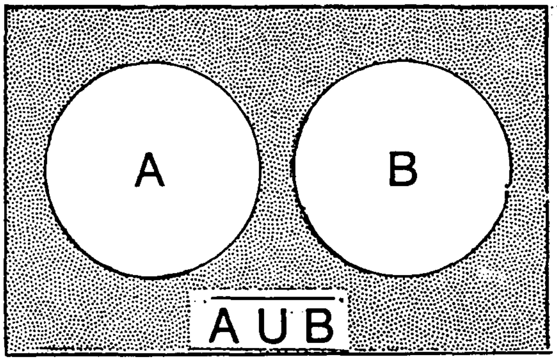
D.2 Exact calculation, JUGREC.



a. Union of Sets A and B.



b. Intersections of Sets A and B.



c. A and B as Mutually Exclusive Sets.

Figure D-1. Venn Diagrams for Any Two Sets.

Figure D-1 shows Venn diagrams of any two sets A and B. In Figure D-1a the sets overlap because they have some elements in common and in D-1c they have no common elements. Every element of the black bordered "universe" is either in "A or B", written $A \cup B$, or is in "not (A or B)", written $\overline{A \cup B}$. In Figure D-1b, the sub-regions of D-1a are identified as elements in one set and not in the other set, $A \cap \bar{B}$ or $B \cap \bar{A}$; the overlap region of A and B is identified as $A \cap B$.

By writing equivalent expressions for the region of $A \cup B$, the very same elements are contained, which makes the sets equal. Inspection of Figure D-1b and D-1c shows that the region identified as $A \cup B$ in D-1a can be written as

$$A \cup B = A + B - (A \cap B), \tag{D13}$$

$$A \cup B = A + (B \cap \bar{A}). \tag{D14}$$

In JUGFAE, a 2-jug line involves just the set E_1, so Equation D1 gives failure

for $n = 2$, $\quad P(f) = P(E_1) = p'$.

For $n = 3$,

$$P(f) = P(E_1 \cup E_2) = P(E_1 + [E_2 \cap \overline{E}_1]), \qquad \text{(D15)}$$

$$= P(E_1) + P(E_2 \cap \overline{E}_1),$$

$$= P(E_1) + P(E_2) = 2p',$$

where Equation D14 and the additive probability of mutually exclusive events and the fact that an element in "not E_1" has to be in E_2 were used. With Equation D15 as a model, the extension to a 4-jug line is done if in place of E_1 we write $E_1 \cup E_2$, and in place of E_2 we write E_3.

For $n = 4$,

$$P(f) = P(E_1 \cup E_2 \cup E_3),$$

$$= P(E_1 \cup E_2) + P(E_3 \cap [\overline{E_1 \cup E_2}]). \qquad \text{(D16)}$$

All manipulations are done on the sets in the second term so we work with just that much then join the intermediate result to Equation D16.

$$E_3 \cap [\overline{E_1 \cup E_2}] = E_3 \cap [\overline{E}_1 \cap \overline{E}_2], \qquad \text{De Morgan's theorem,}$$

$$= E_3 \cap \overline{E}_1 \cap \overline{E}_2,$$

$$= E_3 \cap \overline{E}_1 \cap E_3 \cap \overline{E}_2, \qquad A \cap A = A,$$

$$= [E_3 \cap \overline{E}_1] \cap [E_3 \cap \overline{E}_2], \qquad \text{association,}$$

$$= [E_3 \cap \overline{E}_1] \cap E_3, \qquad E_3 \text{ and } E_2 \text{ do not intersect. } E_3 \text{ is a subset of "not } E_2\text{".}$$

$$= E_3 \cap \overline{E}_1 \cap E_3, \qquad \text{drop braces,}$$

$$E_3 \cap [\overline{E_1 \cup E_2}] = E_3 \cap \overline{E}_1. \qquad \text{per third step above.}$$

So Equation D16 becomes

$$P(f) = P(E_1 \cup E_2) + P(E_3 \cap \overline{E}_1), \qquad \text{(D17)}$$

$$= P(E_1 \cup E_2) + P(E_3)P(\overline{E}_1), \qquad \text{independent events}$$

$$= P(E_1 \cup E_2) + P(E_3)[1 - P(E_1)], \text{ complementarity}$$

$$P(\text{fail w/4 jugs}) = P(\text{fail w/3 jugs}) + p'[1 - P(\text{fail w/2 jugs})]. \qquad \text{(D18)}$$

Equation D18 was explicitly derived because a 4-jug line is the shortest line for which the failure equation is seen as a specialization of the general form. We go on to derive the general form, but the symbology is tedious to follow. The reader may prefer to note that the result, Equation D25, subsumes Equation D18.

Recall from Equation D1 that the probability of failure P(f) for n jugs is

$$P(f) = P(E_1 \cup E_2 \cup \ldots \cup E_{n-2} \cup E_{n-1}),$$

$$= P[(E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2}) \cup E_{n-1}], \quad \text{association}$$

$$= P\{(E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2})$$

$$+ [E_{n-1} \cap (\overline{E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2}})]\}, \quad \text{by Equation D14.}$$

The whole expression inside braces should be thought of as two sets, E_{n-1}, and a set which is all the united sets inside the parentheses; its form is $\{A + B \cap \overline{A}\}$ which is the sum of two sets that are always mutually exclusive (recall Figure D-1). So the probability of occurrence is the sum of the separate probabilities of occurrence, as was already used at Equation D16. Then we can write:

$$P(f) = P(E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2})$$

$$+ P[E_{n-1} \cap (\overline{E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2}})]. \quad \text{(D19)}$$

All manipulations are performed on the sets in the second term, so we work with it, analogously to the three-event manipulations after Equation D16, then join the intermediate result to Equation D19:

$$E_{n-1} \cap [(\overline{E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2}})] = [E_{n-1} \cap (\overline{E}_1 \cap \overline{E}_2 \cap \ldots \quad \text{(D20)}$$

$$\cap \overline{E}_{n-3} \cap \overline{E}_{n-2})], \quad \text{De Morgan}$$

$$= E_{n-1} \cap \overline{E}_1 \cap \overline{E}_2 \cap \ldots \cap \overline{E}_{n-3} \cap \overline{E}_{n-2}, \quad \text{drop parentheses}$$

$$= E_{n-1} \cap \overline{E}_1 \cap E_{n-1} \cap \overline{E}_2 \cap \ldots \cap E_{n-1} \cap \overline{E}_{n-3} \cap E_{n-1} \cap \overline{E}_{n-2}, \quad A \cap B \cap C = A \cap B \cap A \cap C$$

$$= (E_{n-1} \cap \bar{E}_1) \cap (E_{n-1} \cap \bar{E}_2) \cap \ldots \cap (E_{n-1} \cap \bar{E}_{n-3}) \cap (E_{n-1} \cap \bar{E}_{n-2})$$

$$= (E_{n-1} \cap \bar{E}_1) \cap (E_{n-1} \cap \bar{E}_2) \cap \ldots \cap (E_{n-1} \cap \bar{E}_{n-3}) \cap E_{n-1} \,,$$

where the last parentheses are dropped because $E_{n-1} \cap \bar{E}_{n-2} = E_{n-1}$ since E_{n-1} and $\bar{E}_{n-2}$ do not intersect,

$$= (E_{n-1} \cap \bar{E}_1) \cap (E_{n-1} \cap \bar{E}_2) \cap \ldots \cap (E_{n-1} \cap \bar{E}_{n-3}) \,,$$

where the last E_{n-1} is dropped because it is already intersected.
For the same reason as previously stated, all but one E_{n-1} can be dropped.

$$= E_{n-1} \cap \bar{E}_1 \cap \bar{E}_2 \cap \ldots \cap \bar{E}_{n-3} \,,$$

$$= E_{n-1} \cap (\bar{E}_1 \cap \bar{E}_2 \cap \ldots \cap \bar{E}_{n-3}) \,, \quad \text{association}$$

$$= E_{n-1} \cap (\overline{E_1 \cup E_2 \cup \ldots \cup E_{n-3}}) \,, \quad \text{De Morgan}$$

So Equation D20 becomes,

$$(E_{n-1} \cap [(\overline{E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2}})] = E_{n-1} \cap (\overline{E_1 \cup E_2 \cup \ldots \cup E_{n-3}}). \tag{D21}$$

Equation D21 is substituted into Equation D19 and it becomes,

$$P(f) = P(E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2}) + P(E_{n-1} \cap [(\overline{E_1 \cup E_2 \cup \ldots \cup E_{n-3}})]. \tag{D22}$$

The sets in the second term are independent and since $P(A \cap B) = P(A)P(B)$ when A and B are independent,

$$P(f) = P(E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2}) + P(E_{n-1})[P(\overline{E_1 \cup E_2 \cup \ldots \cup E_{n-3}})], \tag{D23}$$

by complementarity,

$$P(f) = P(E_1 \cup E_2 \cup \ldots \cup E_{n-3} \cup E_{n-2}) + P(E_{n-1})[(1 - P(E_1 \cup E_2 \cup \ldots \cup E_{n-3})], \tag{D24}$$

Equation D24 is analogous to the equation before (D18) and with that one and Equation D1, we can write,

$$P(\text{fail w/n jugs}) = P(\text{fail w/n-1 jugs}) + (p')[1 - P(\text{fail w/n-2 jugs})], \tag{D25}$$

where $p' = (p/2)^2$, p = probability of one jug being mislaid.

Equation D25 is the basis of the program JUGREC.

TABLE D2. JUGREC Program.

```
      PROGRAM JUGREC
C     PROGRAM TO CALCULATE THE PROBABILITY OF FAILURE TO
C     DETONATE DUE TO CRANE ERROR (MISPLACED JUGS)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      WRITE(6,1)
    1 FORMAT(/'CALCULATION OF PROBABILITY OF FAILURE TO DETONATE'/
     *'DUE TO CRANE ERROR (MISPLACED JUGS)'//
     *'WHEN PROMPTED, ENTER N, THE NUMBER OF JUGS IN A ROW,'/
     *'AND P, THE PROBABILITY OF AN INDIVIDUAL JUG BEING LAID'/
     *'OUTSIDE ITS ERROR CIRCLE.'//
     *'N MUST BE A POSITIVE INTEGER GREATER THAN OR EQUAL TO 2,'/
     *'AND P MUST BE GREATER THAN OR EQUAL TO 0 AND'/
     *'LESS THAN OR EQUAL TO 1.'//
     *'ENTER ZEROES FOR N AND P WHEN NO MORE DATA ARE TO BE ENTERED.'/)
    2 READ(5,*)N,P
      IF(N.EQ.0)GO TO 5
      P2=(P/2)**2
      PRFAIL2=0
      PRFAIL1=0
      PRFAIL=0
      DO 3 I=2,N
      PRFAIL2=PRFAIL1
      PRFAIL1=PRFAIL
    3 PRFAIL=PRFAIL1+P2*(1-PRFAIL2)
      WRITE(6,4)N,P,PRFAIL
    4 FORMAT(/'N =',T16,I4/
     *'P =',T14,F6.4//
     *'PROBABILITY'/
     *'OF FAILURE = ',F6.4/)
      GO TO 2
    5 STOP
      END
```

D.3 Approximate calculation. A quick and easy approximation can be used if we proceed as if all of the events E_1, E_2, ..., E_{n-1} were independent.

As before in Equation D1, for an n-jug line the probability of failure is

$$P(f) = P(E_1 \cup E_2 \cup \ldots \cup E_{n-1}),$$

$$= 1 - P(\overline{E_1 \cup E_2 \cup \ldots \cup E_{n-1}}), \qquad \text{complementarity} \qquad (D26)$$

$$= 1 - P(\overline{E}_1 \cap \overline{E}_2 \cap \ldots \cap \overline{E}_{n-1}), \qquad \text{De Morgan's theorems} \qquad (D27)$$

$$\simeq 1 - P(\overline{E}_1)P(\overline{E}_2) \ldots P(\overline{E}_{n-1}), \qquad \text{incorrectly assumes all events are independent} \qquad (D28)$$

$$\simeq 1 - \{[1-P(E_1)][1-P(E_2)] \ldots [1-P(E_{n-1})]\}, \qquad \text{complementarity} \qquad (D29)$$

$$\simeq 1 - (1-p')^{n-1}, \qquad p' = (p/2)^2 \qquad (D30)$$

$$P(f) \simeq 1 - [1 - (p/2)^2]^{n-1}, \qquad (D31)$$

where p is the probability of mislaying one jug.

This approximation tends to underestimate the probability of failure. For large values of p (close to one) the error is substantial for some values of n but, for the value of p considered in this work ($p = 0.135$), the error will not exceed 0.001 for any value of n. High values of p are not plausible anyway, as JUGFAE could not succeed. The approximation in Equation D31 would be suitable if some quick calculations were needed in the field, where sophisticated computing facilities are not available.

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